

Lecture 7
2018/2019

Microwave Devices and Circuits for Radiocommunications

2018/2019

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course + laboratory)
- Lectures- **associate professor Radu Damian**
 - Friday 09-11, 11.13
 - E – 50% final grade
 - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
 - 3p=+0.5p
 - all materials/equipments authorized
- Laboratory – **associate professor Radu Damian**
 - Wednesday 12-14, 11.12 odd weeks
 - L – 25% final grade
 - P – 25% final grade

Materials

■ <http://rf-opto.etti.tuiasi.ro>

Laboratorul de Microunde si Opti X

Not secure | rf-opto.etti.tuiasi.ro/microwave_cd.php?chg_lang=0

Main **Courses** Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, Sem. 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Examen**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

[Aggregate Results](#)

Attendance

[Course](#)
[Laboratory](#)

Lists

[Bonus-uri acumulate \(final\)](#)
[Studenti care nu pot intra in examen](#)

Materials

Course Slides

[MDCR Lecture 1](#) (pdf, 5.43 MB, en, [ps](#))
[MDCR Lecture 2](#) (pdf, 3.67 MB, en, [ps](#))
[MDCR Lecture 3](#) (pdf, 4.76 MB, en, [ps](#))
[MDCR Lecture 4](#) (pdf, 5.58 MB, en, [ps](#))

Examen: Logarithmic scales

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 μ W
-30 dBm	= 1 μ W
-60 dBm	= 1 nW

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm/Hz}] + [\text{dB}] = [\text{dBm/Hz}]$$

$$[x] + [\text{dB}] = [x]$$

Examen

- Complex numbers arithmetic!!!!
- $z = a + j \cdot b ; j^2 = -1$

Impedance Matching

Matching , from the point of view of power transmission

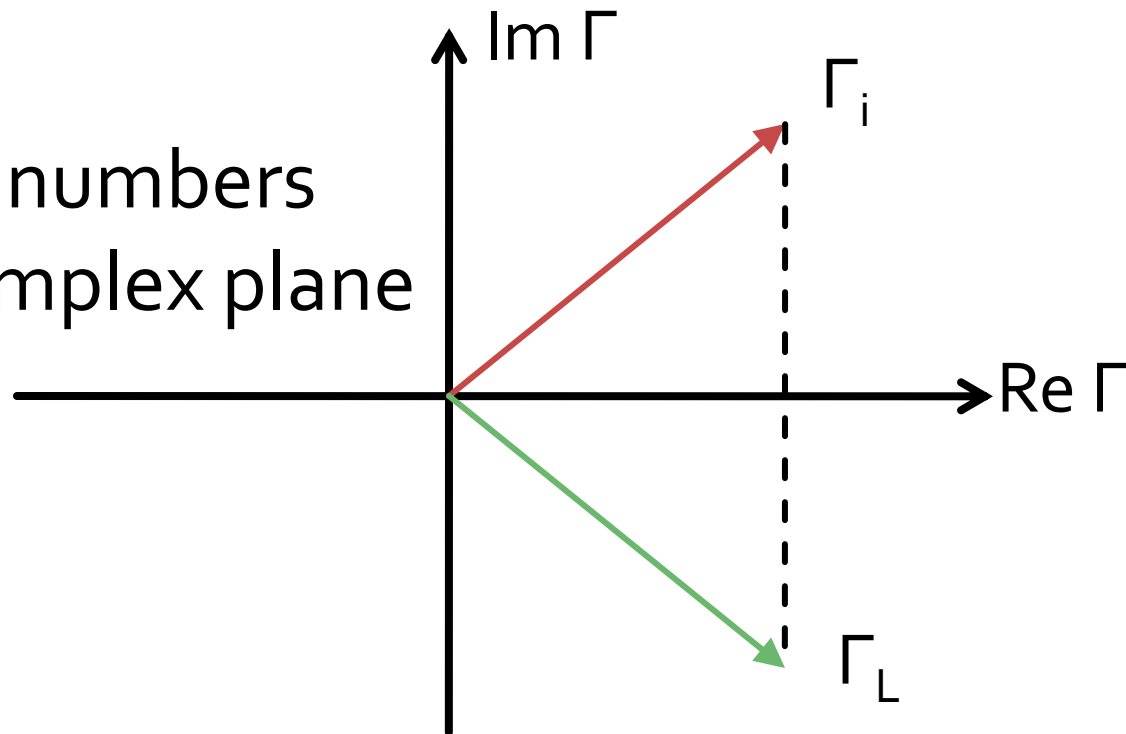
$$Z_L = Z_i^*$$

If we choose a real Z_0

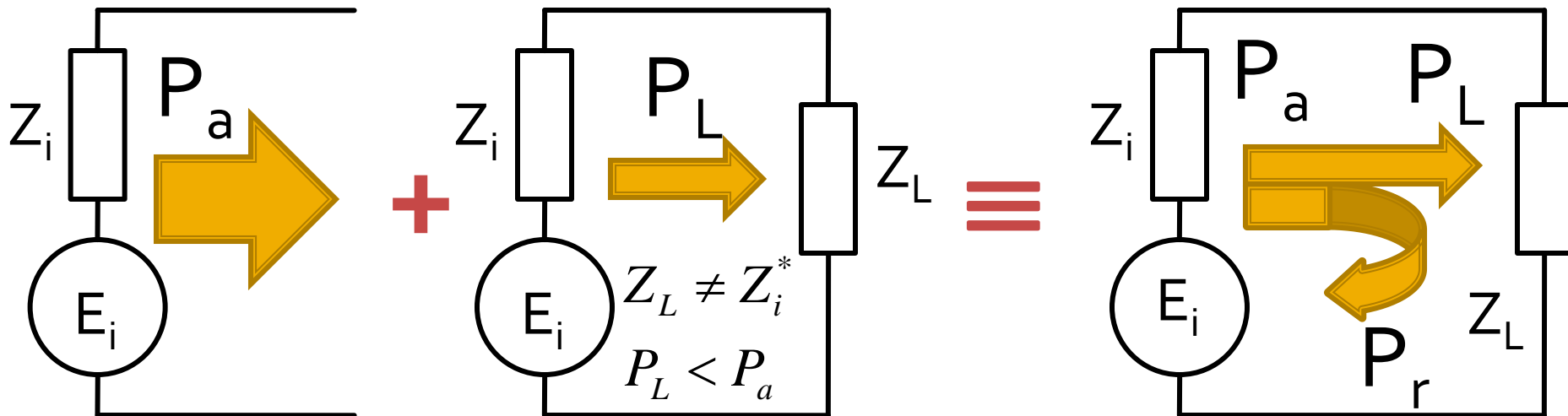
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



Reflection and power / Model



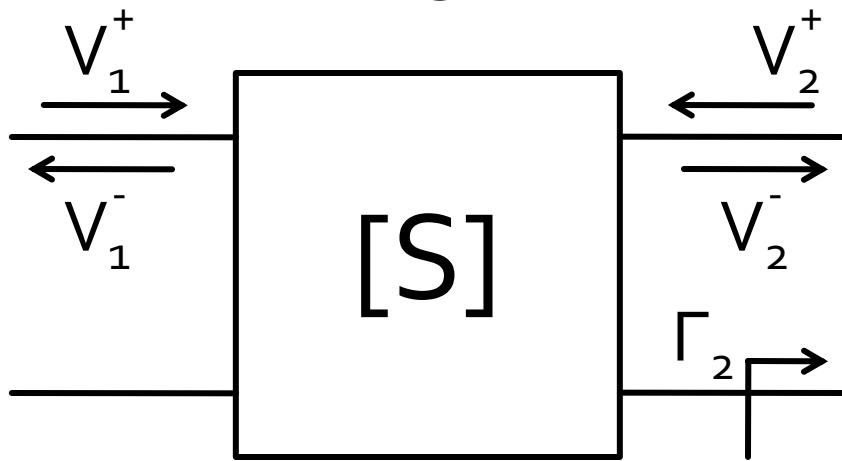
- The source has the ability to send to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar** !

Lecture 3-4

Microwave Network Analysis

Scattering matrix – S

- Scattering parameters



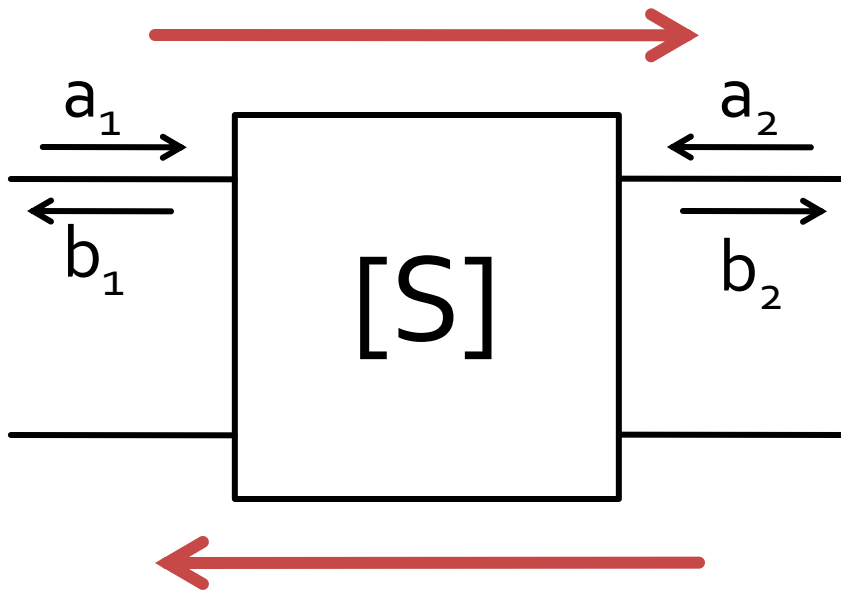
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} \quad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

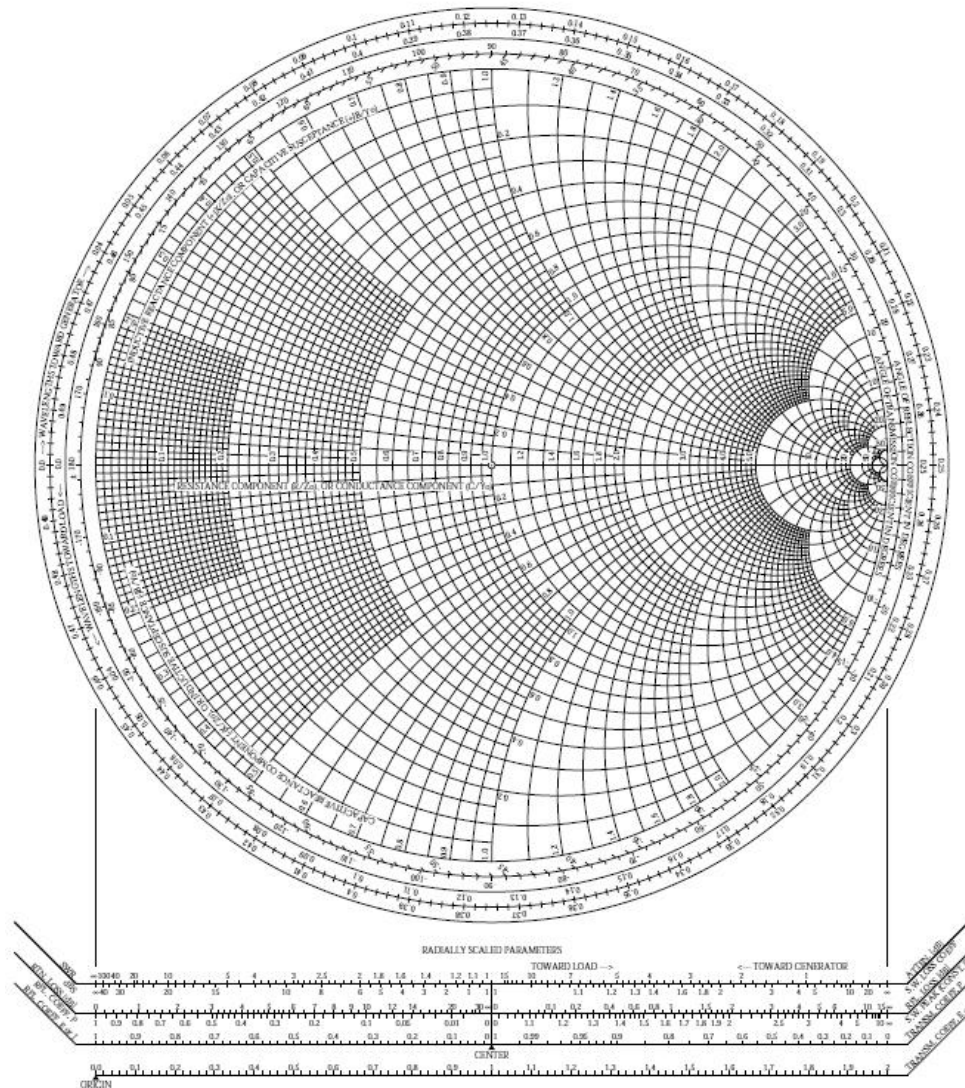
$$|S_{21}|^2 = \frac{\text{Power in } Z_0 \text{ load}}{\text{Power from } Z_0 \text{ source}}$$

- a, b
 - information about signal power **AND** signal phase
- S_{ij}
 - network effect (gain) over signal power **including** phase information

Impedance Matching

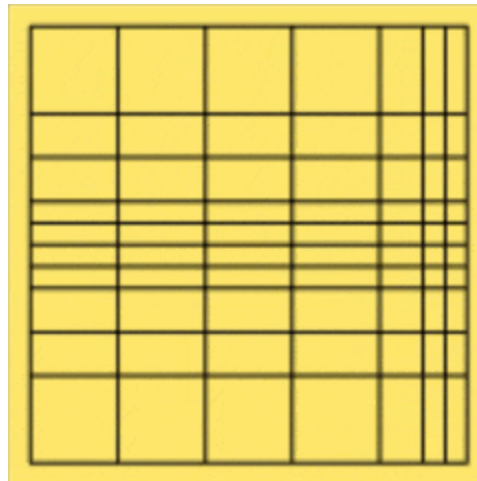
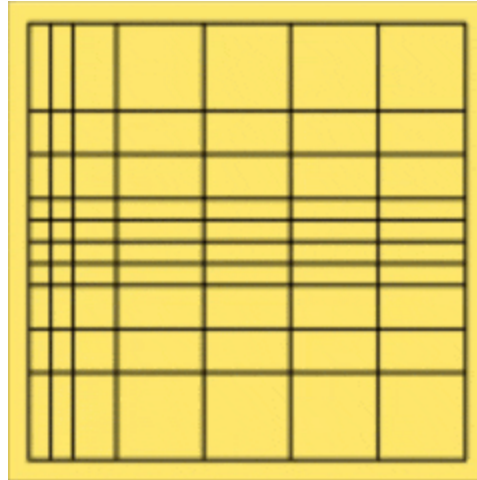
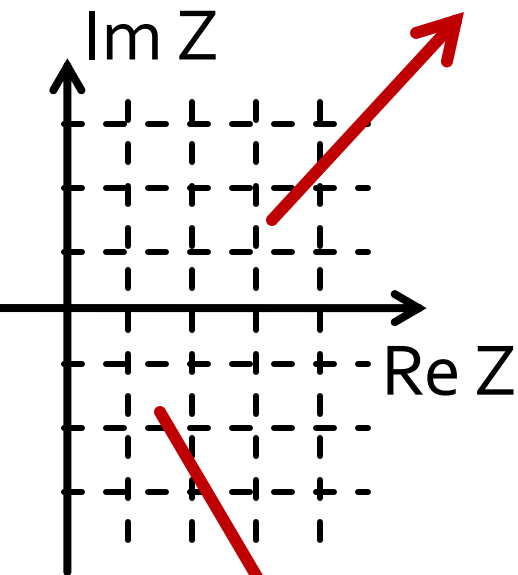
The Smith Chart

The Smith Chart

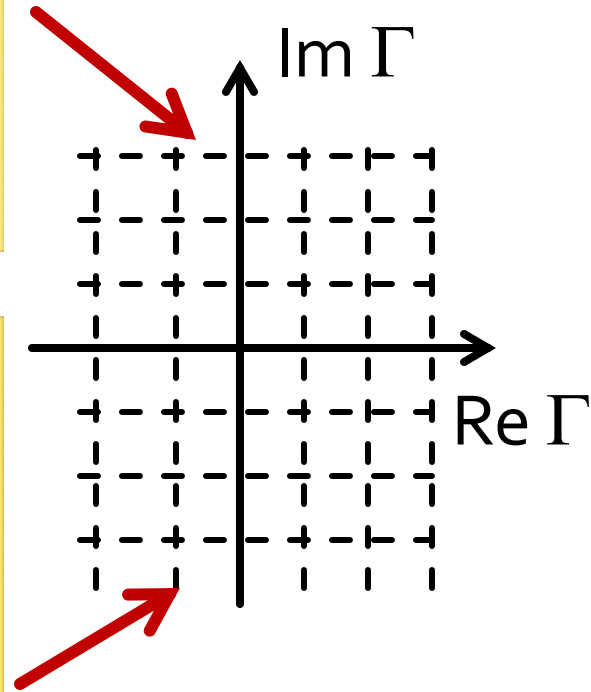


The Smith Chart

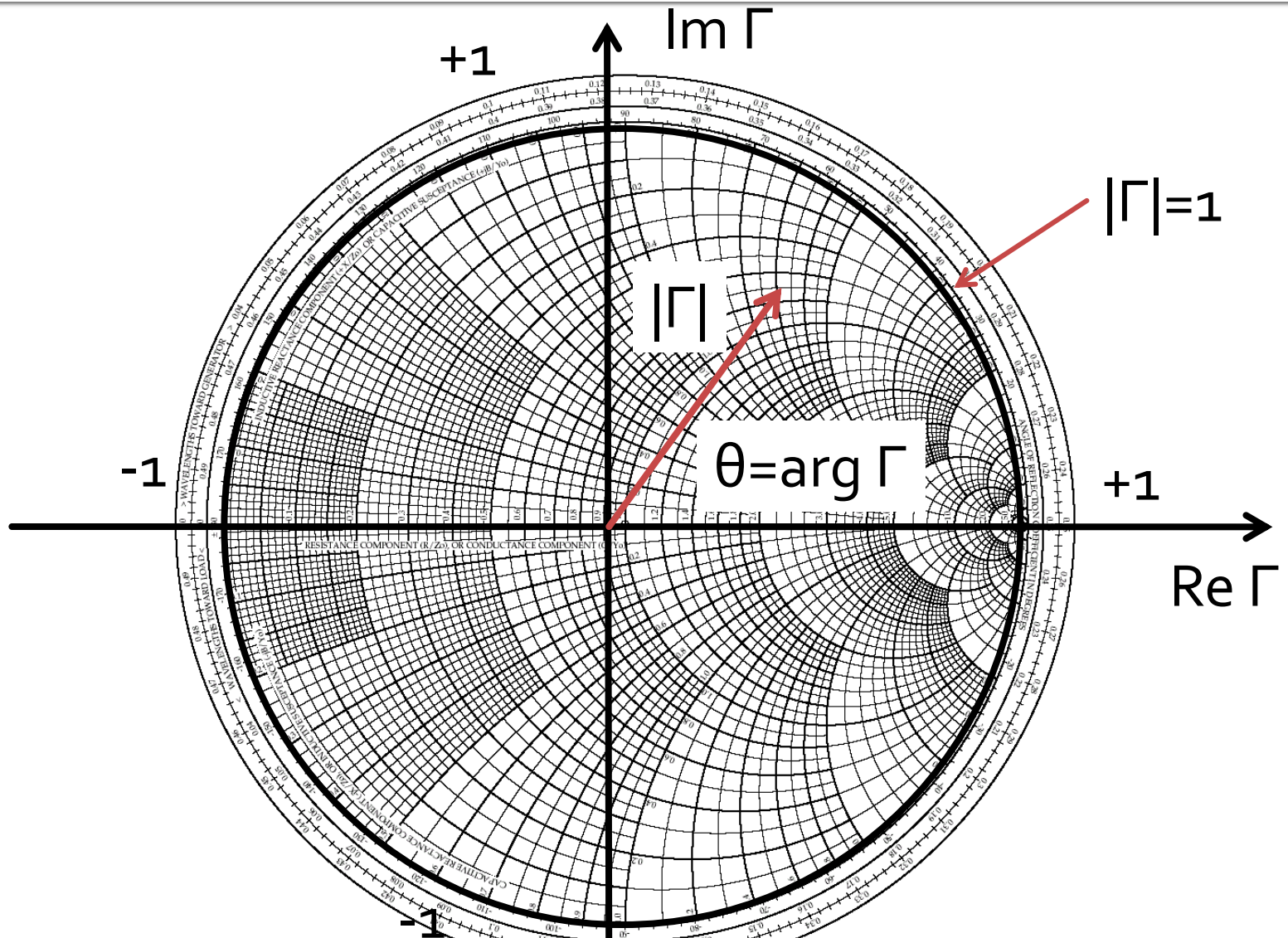
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$



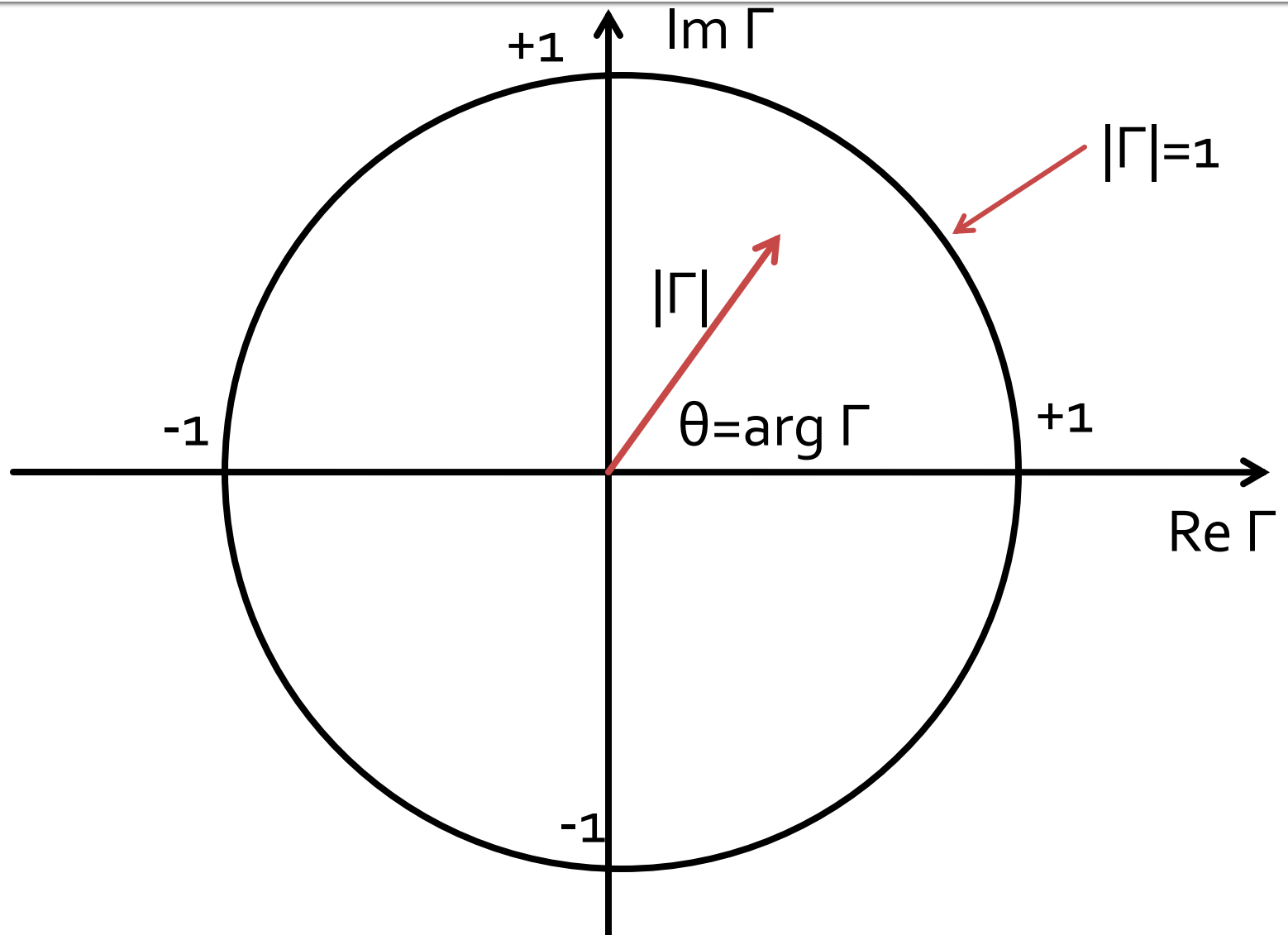
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$



The Smith Chart



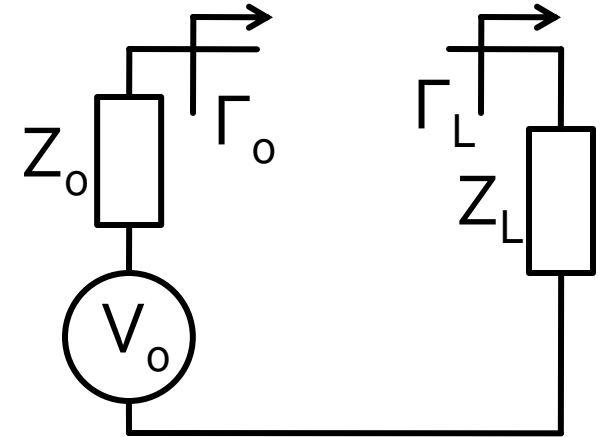
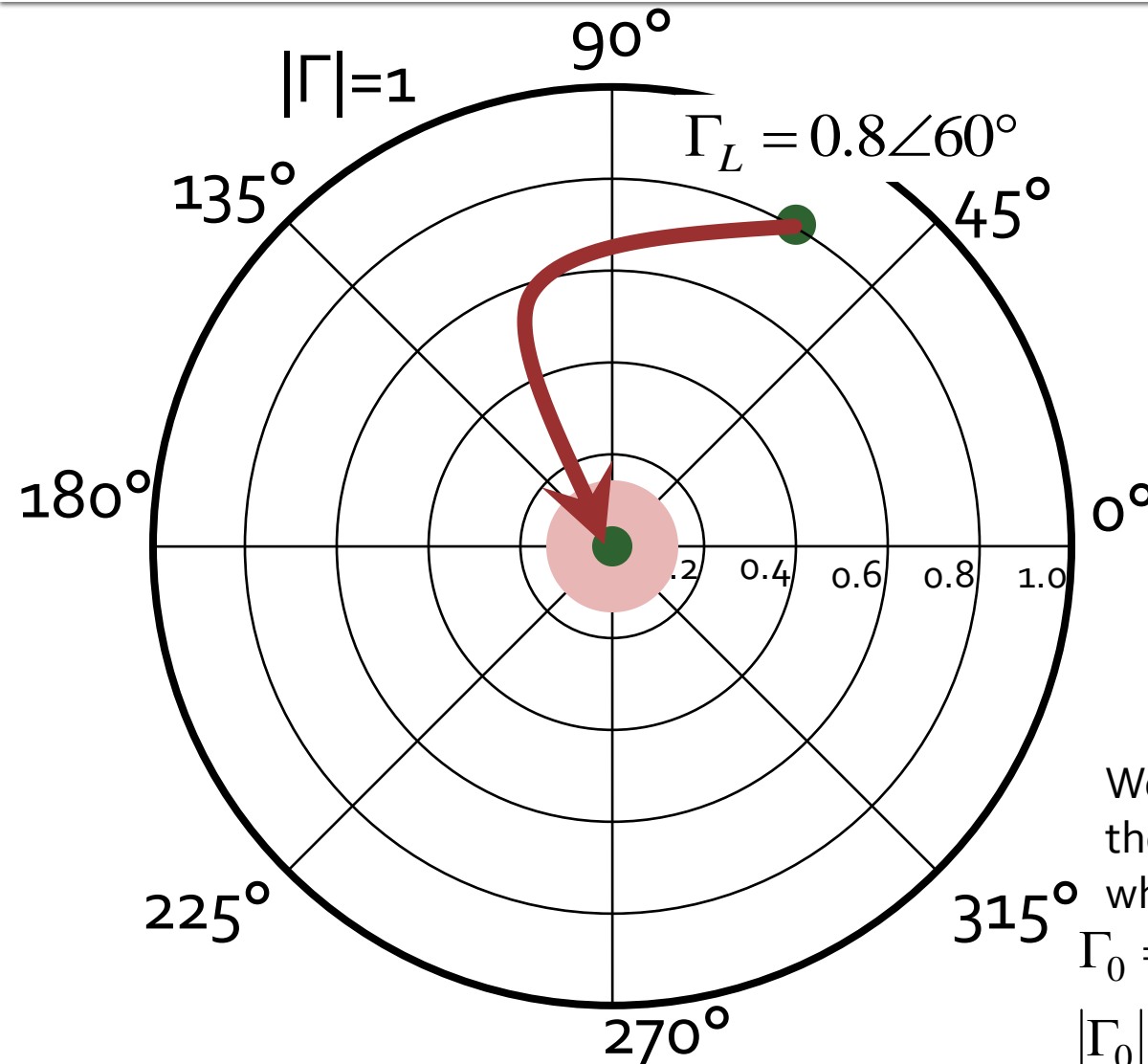
The Smith Chart



Impedance matching with lumped elements (L Networks)

Impedance Matching

The Smith Chart, reflection coefficient, impedance matching



Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

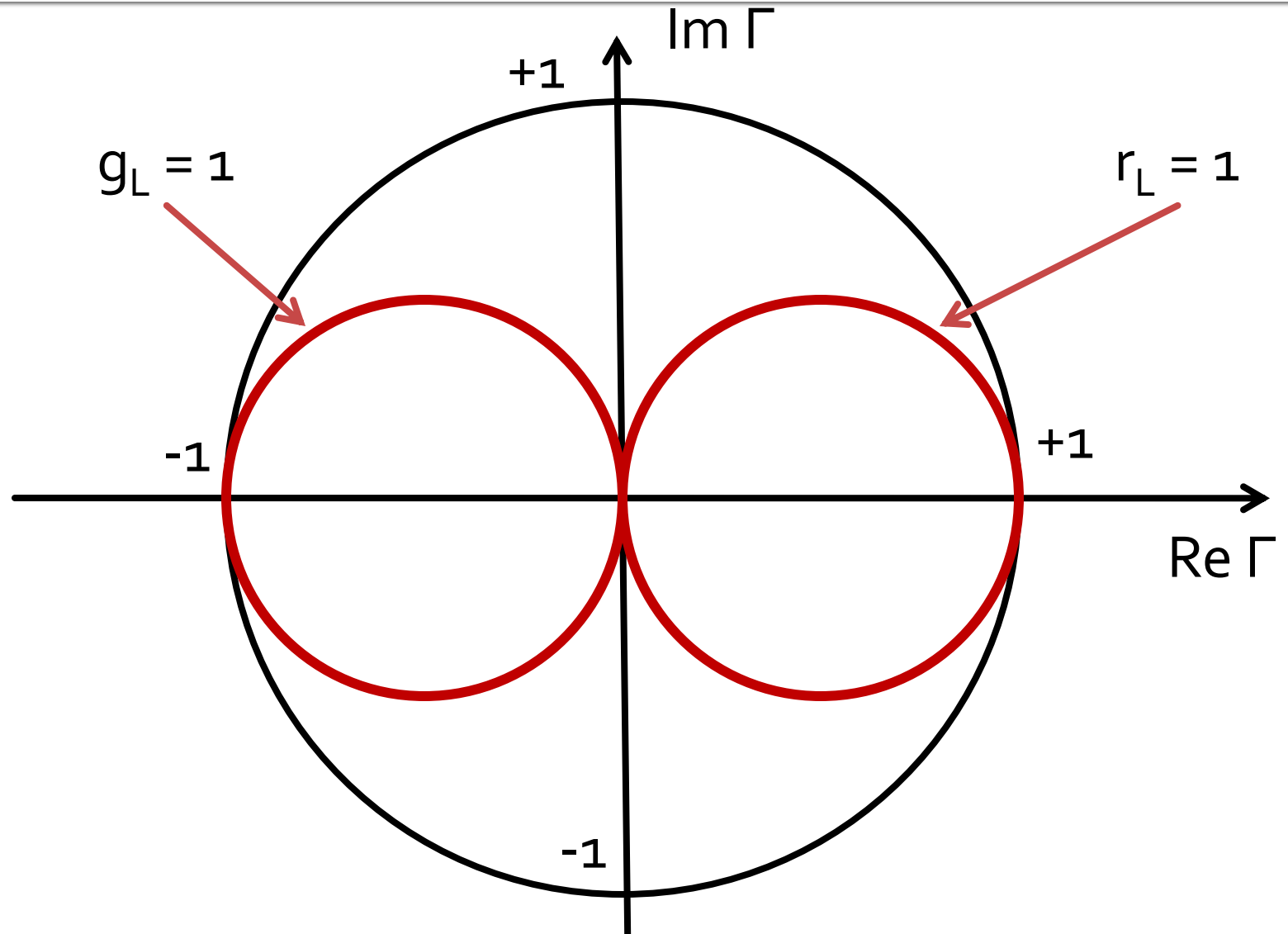
$$\Gamma_L = 0.8 \angle 60^\circ$$

We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

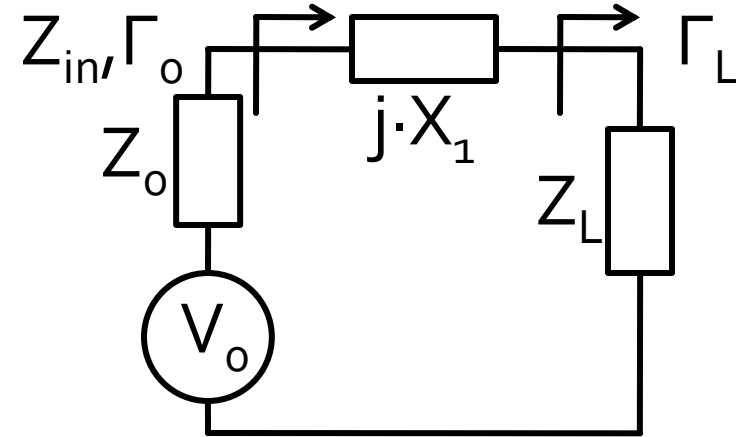
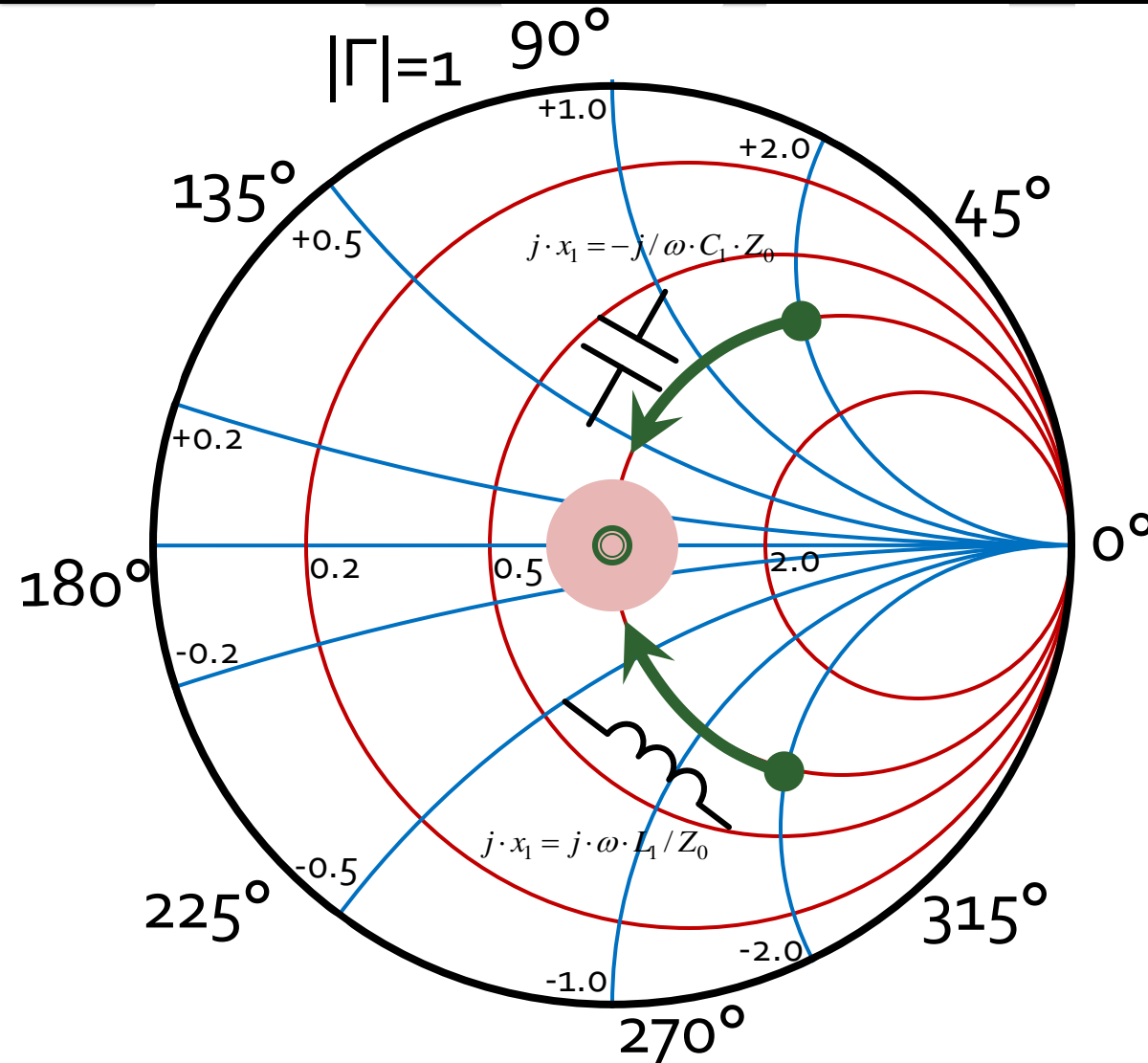
$$\Gamma_o = 0 \text{ perfect match } \bullet$$

$$|\Gamma_o| \leq \Gamma_m \text{ "good enough" match } \bullet$$

Smith chart, $r=1$ and $g=1$



Matching, series reactance



$$z_L = r_L + j \cdot x_L$$

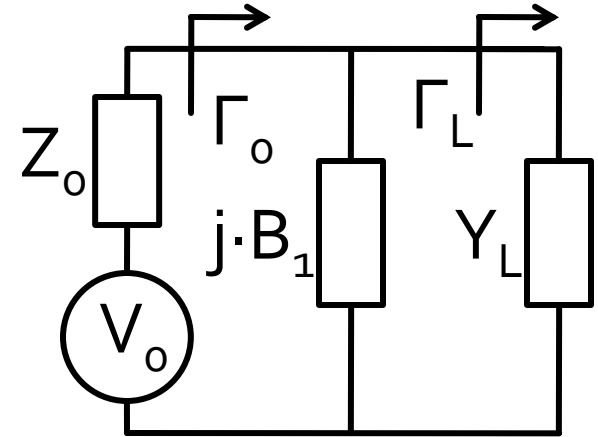
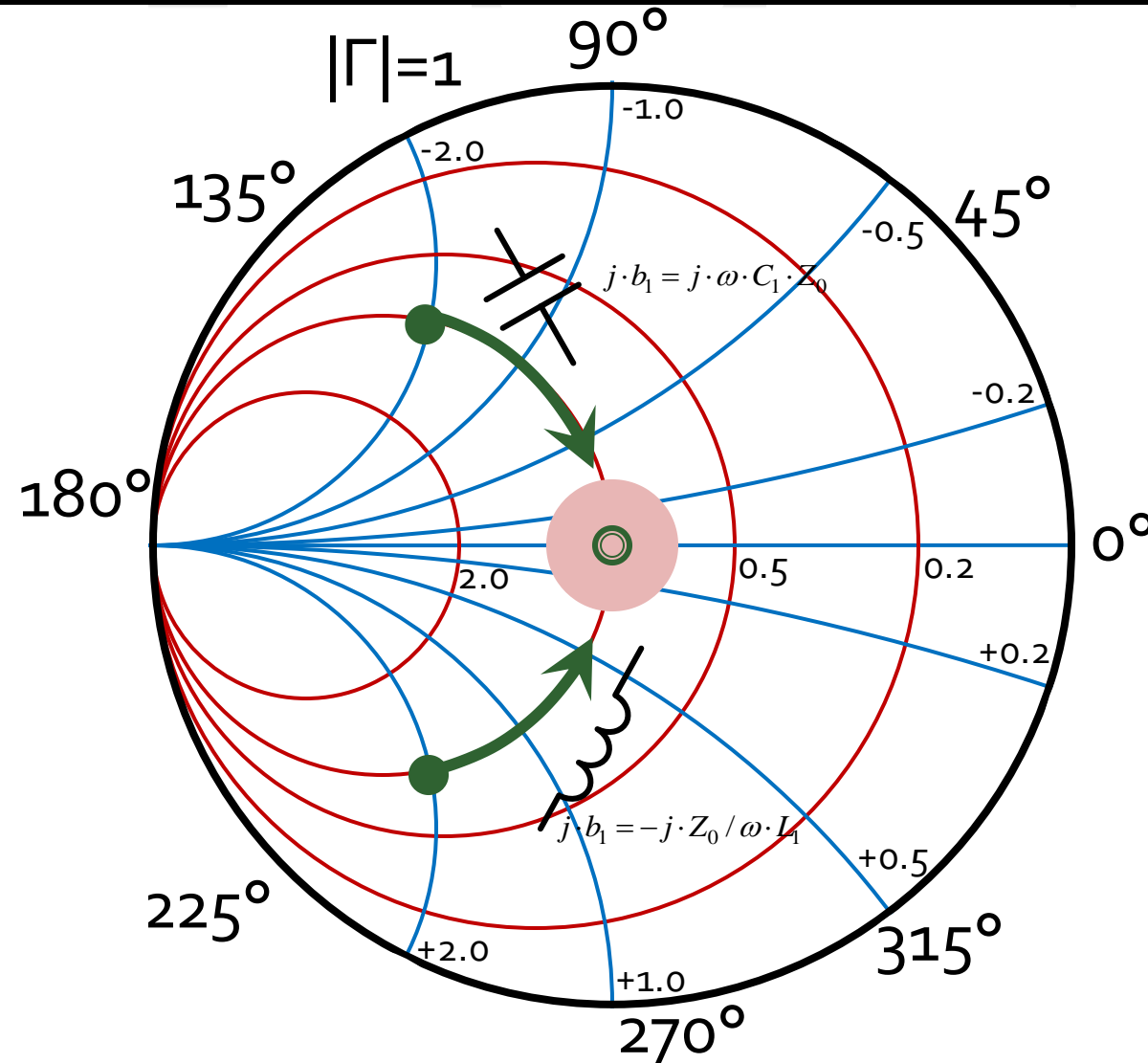
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

- Match can be obtained **if and only if** $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

Matching, shunt susceptance



$$y_L = g_L + j \cdot b_L$$

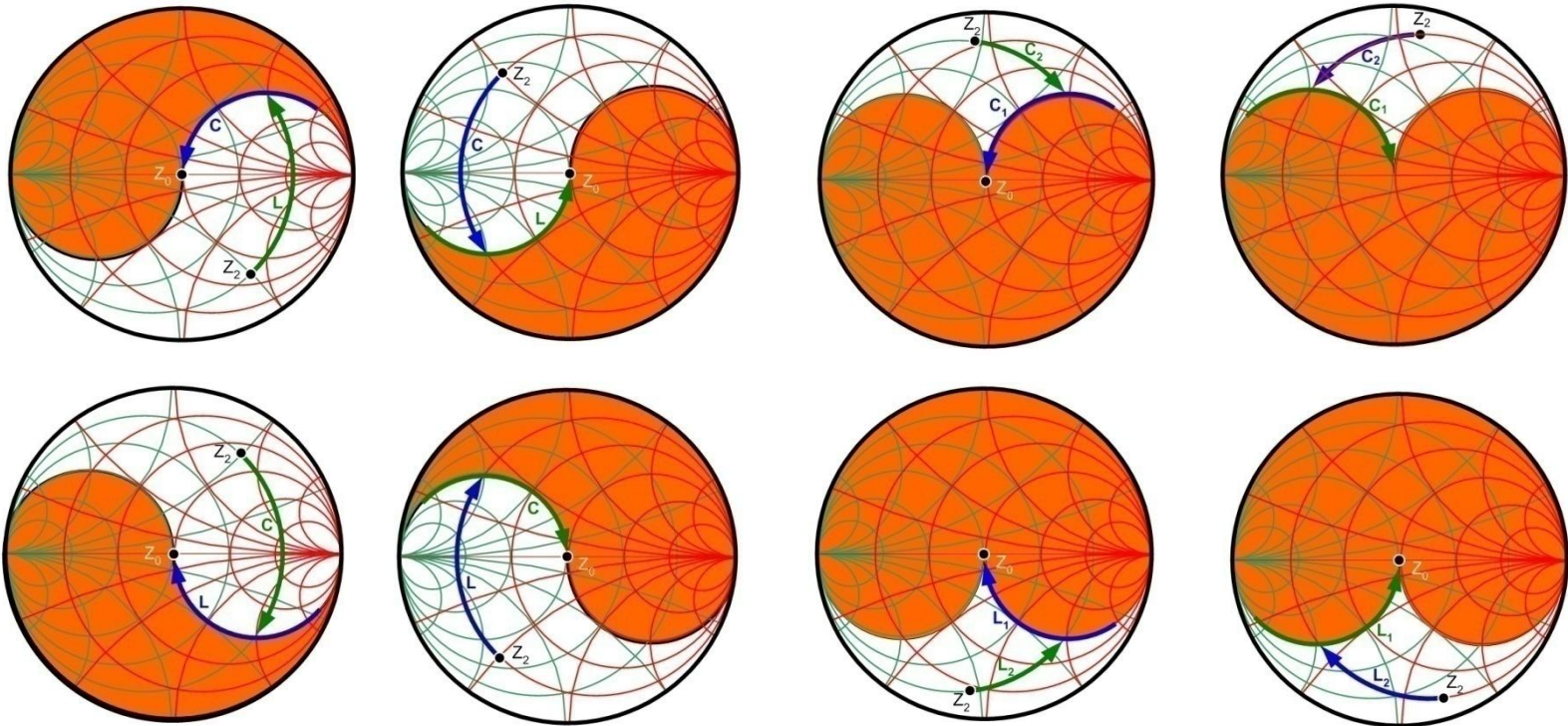
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

$$g_{in} = g_L$$

- Match can be obtained **if and only if** $g_L = 1$
- we compensate the reactive part of the load

$$j \cdot b_1 = -j \cdot b_L$$

Matching with 2 reactive elements (L Networks)

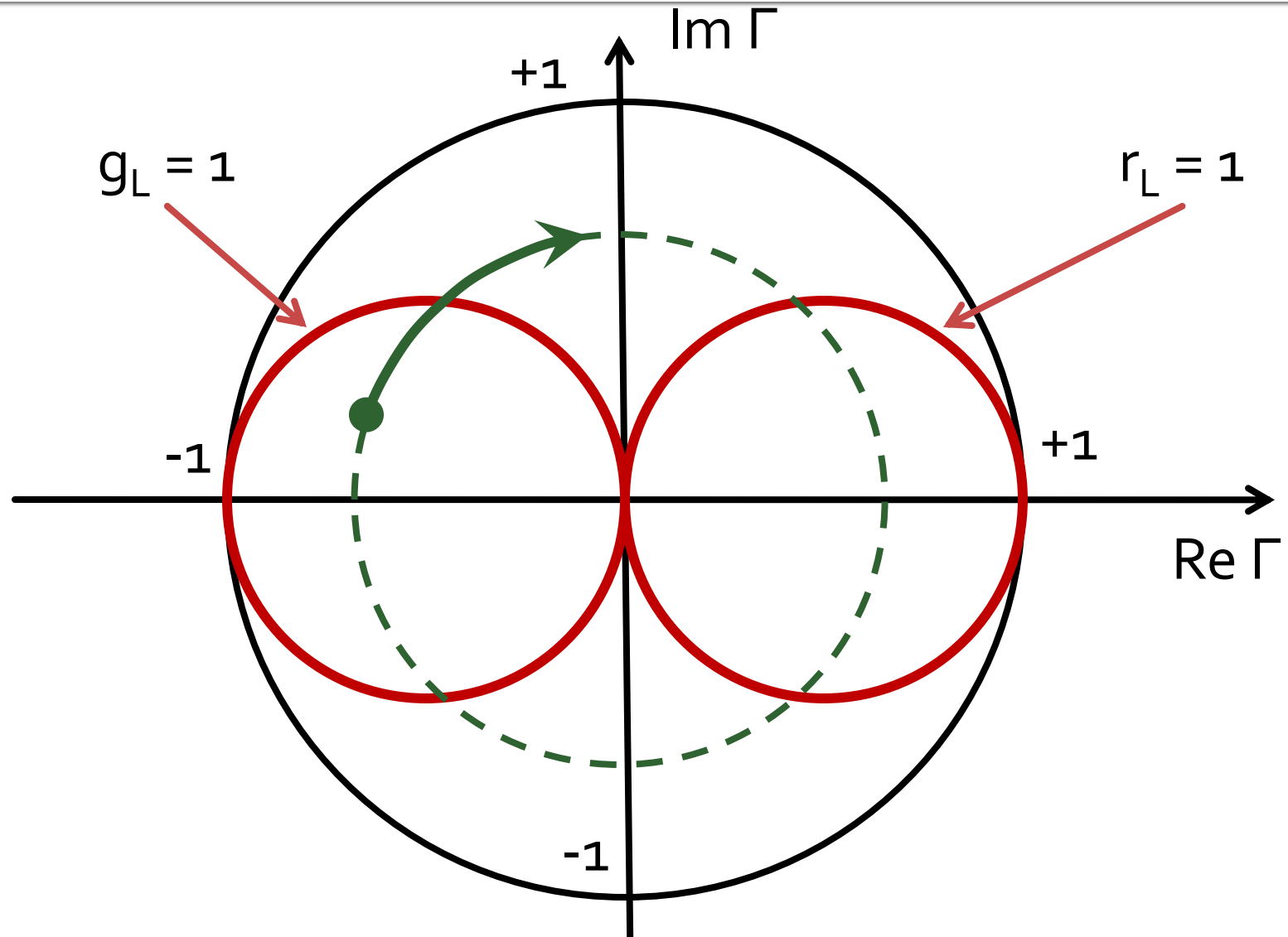


Forbidden area for
current network

Impedance Matching with Stubs

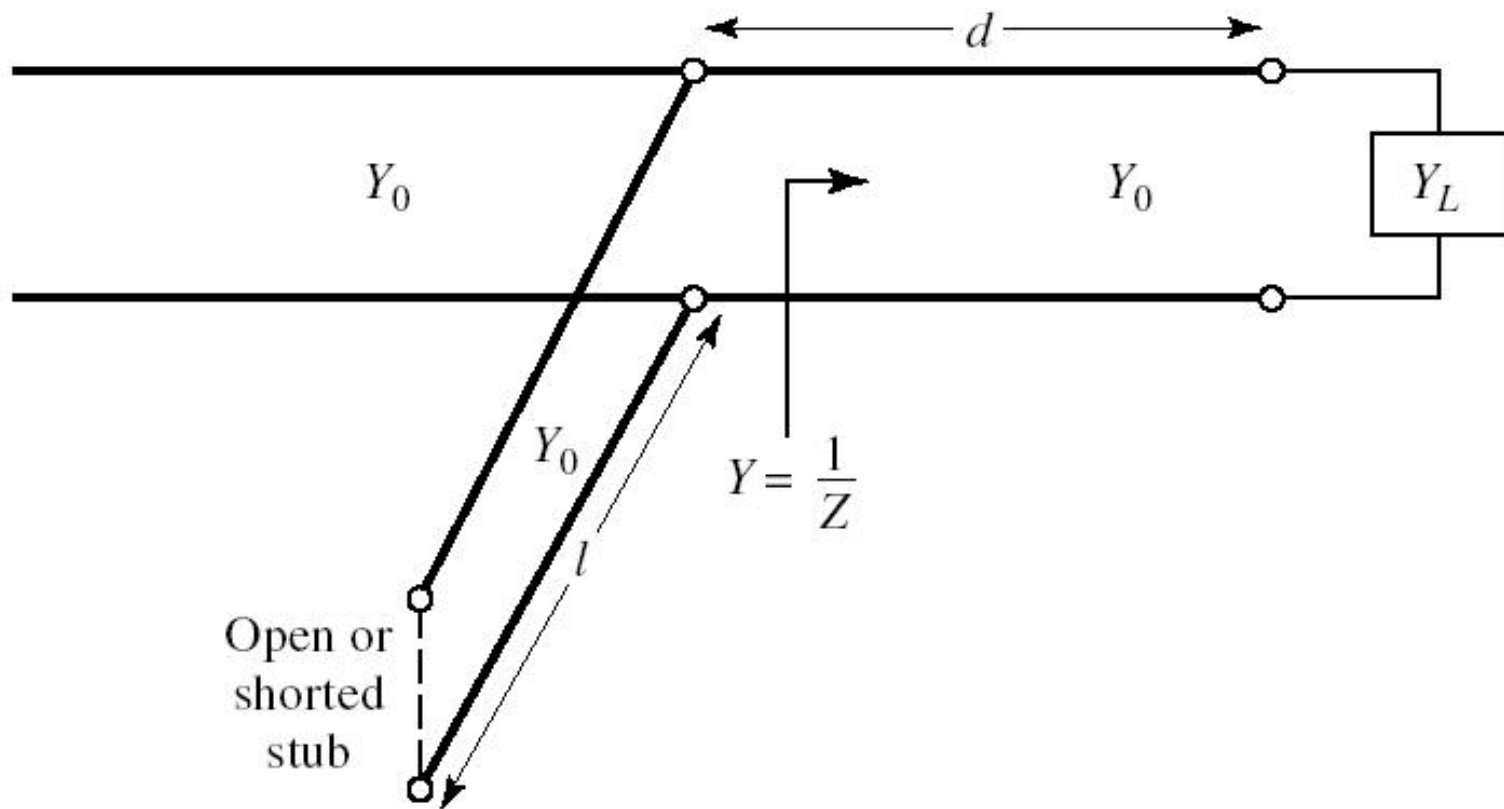
Impedance Matching

Smith chart, $r=1$ and $g=1$



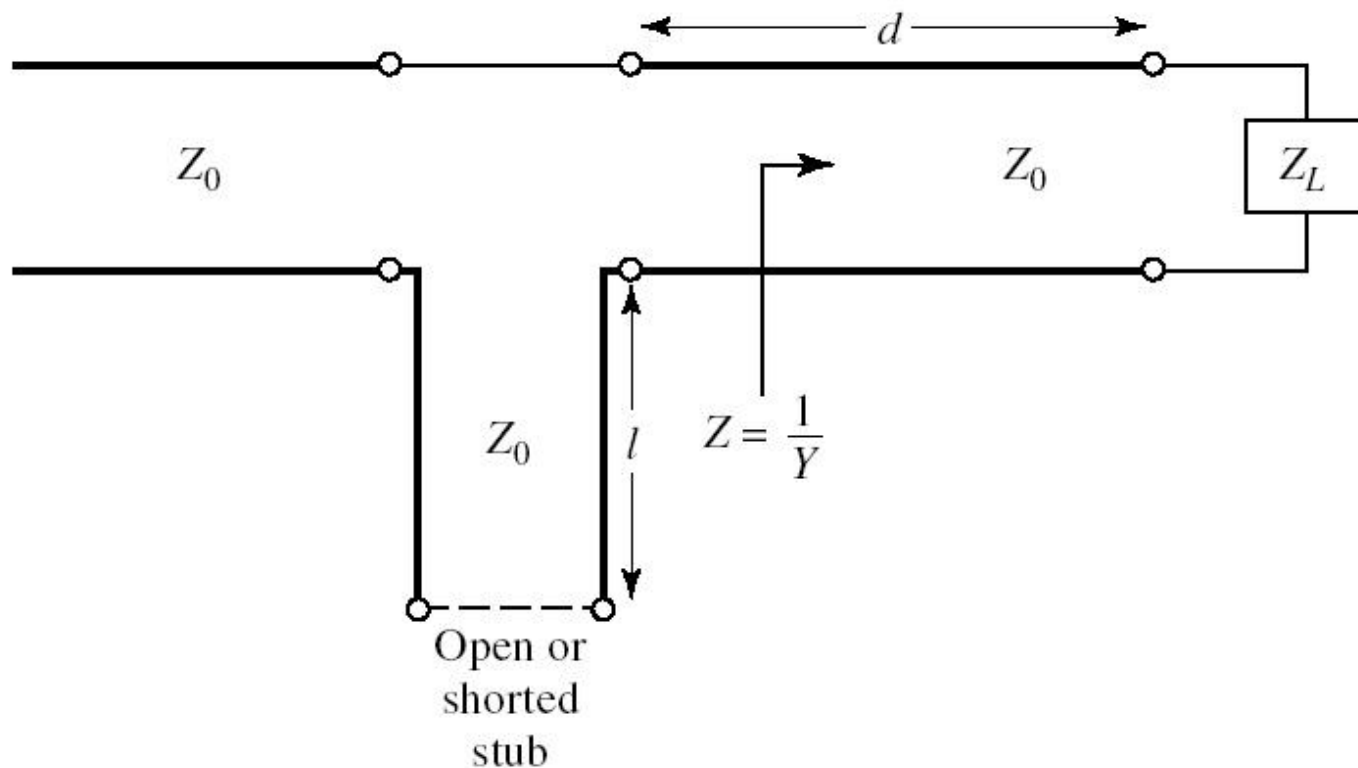
Single stub tuning

- Shunt Stub



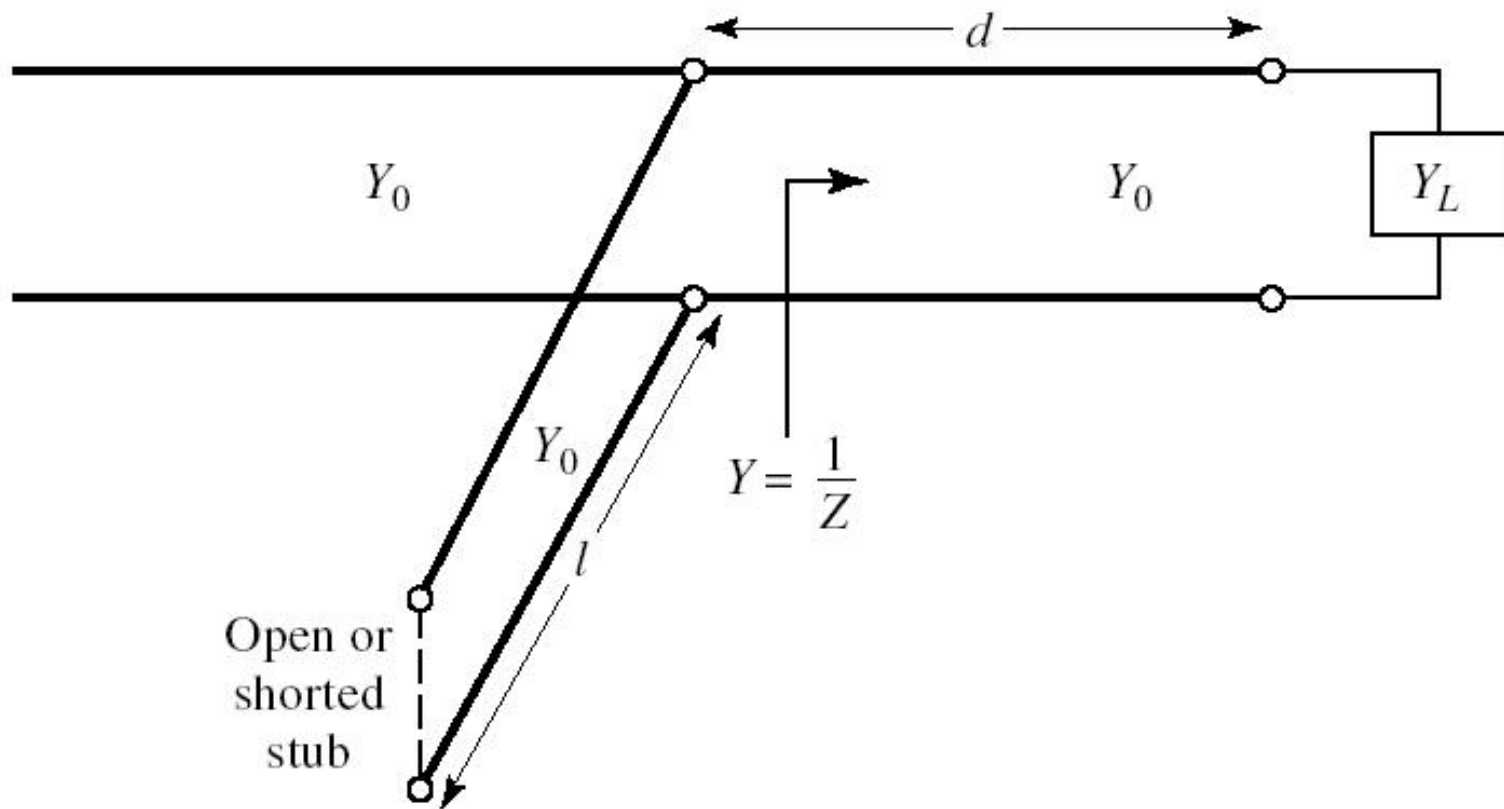
Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)

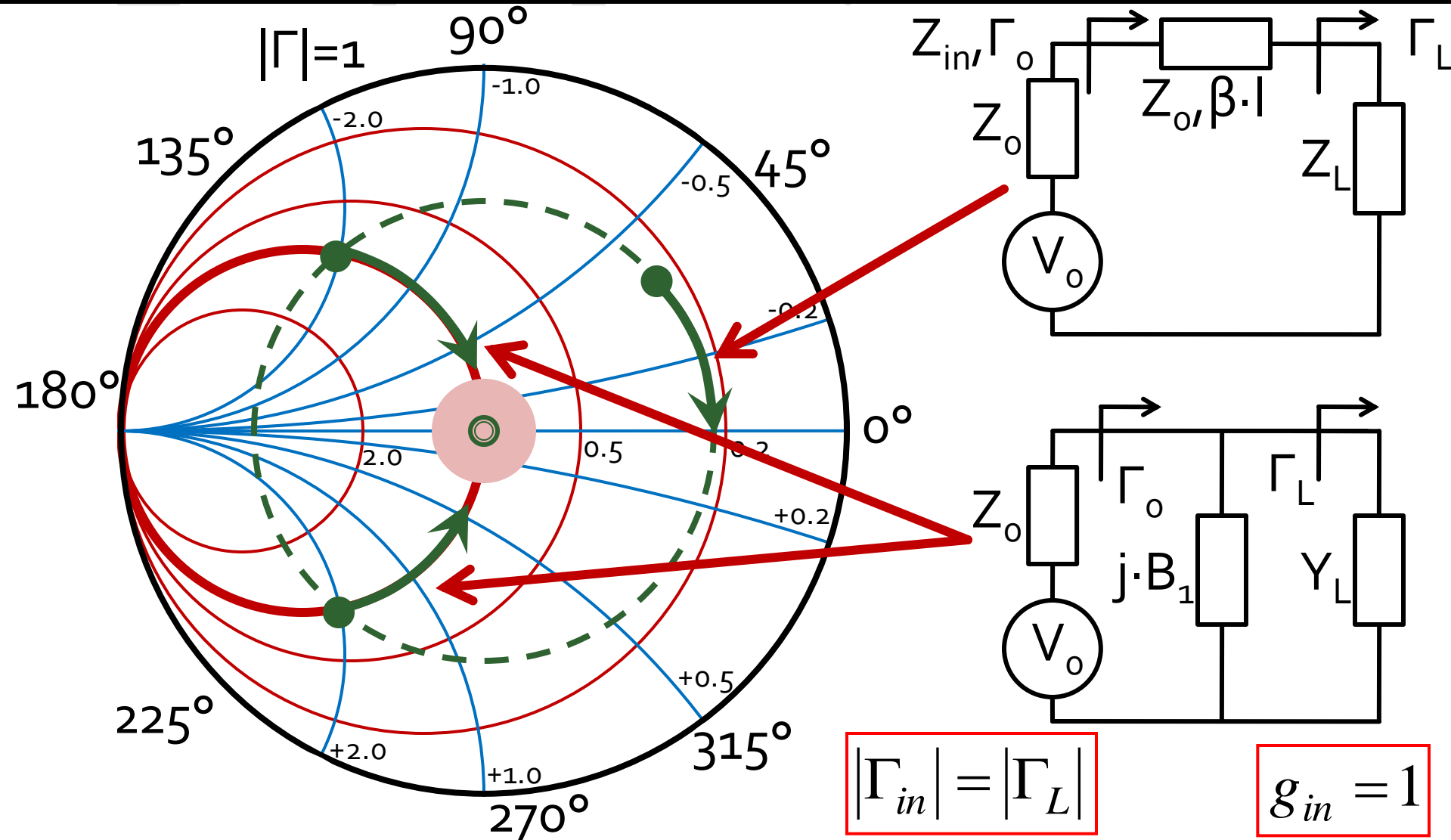


Case 1, Shunt Stub

- Shunt stub



Matching, series line + shunt susceptance



Analytical solution, Γ , shunt stub

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\Gamma_S = 0.593 \angle 46.85^\circ$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85^\circ \quad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

- **“+” solution** ↓

$$(46.85^\circ + 2\theta) = +126.35^\circ \quad \theta = +39.7^\circ \quad \text{Im } y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

$$\theta_{sp} = \tan^{-1}(\text{Im } y_S) = -55.8^\circ (+180^\circ) \rightarrow \theta_{sp} = 124.2^\circ$$

- **“-” solution** ↓

$$(46.85^\circ + 2\theta) = -126.35^\circ \quad \theta = -86.6^\circ (+180^\circ) \rightarrow \theta = 93.4^\circ$$

$$\text{Im } y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \quad \theta_{sp} = \tan^{-1}(\text{Im } y_S) = 55.8^\circ$$

Analytical solution, Γ

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \text{Im}[y_s(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

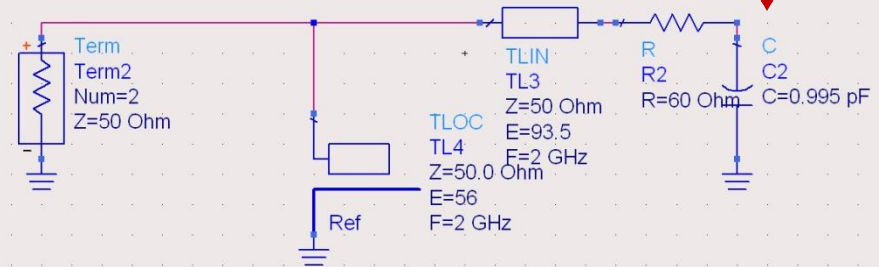
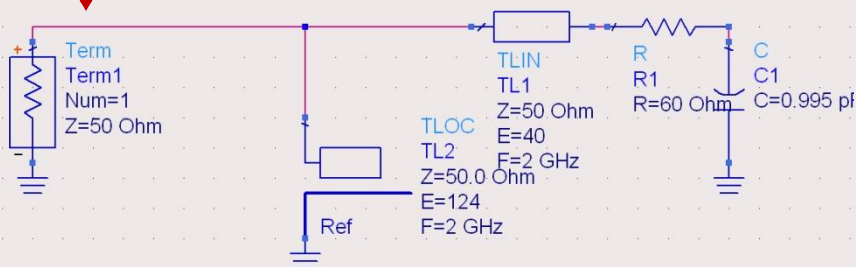
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

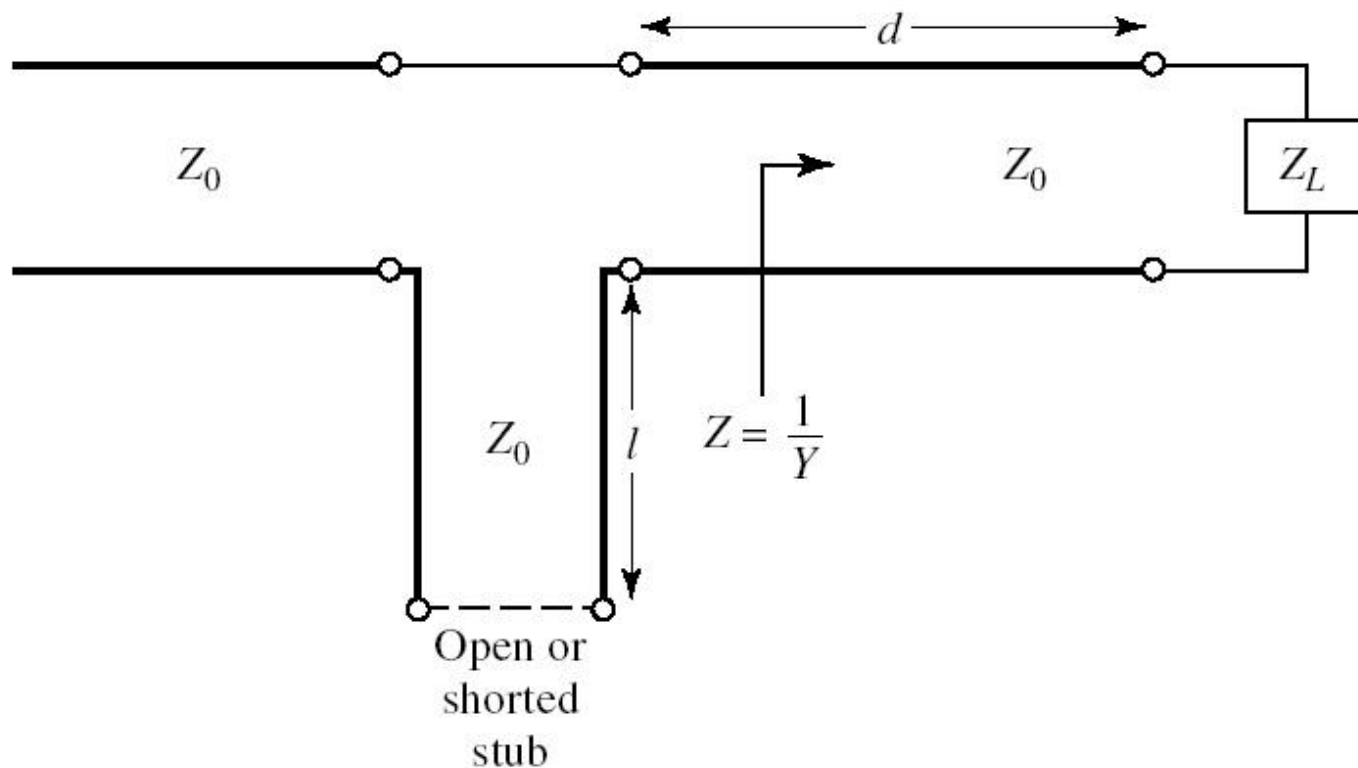
$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$

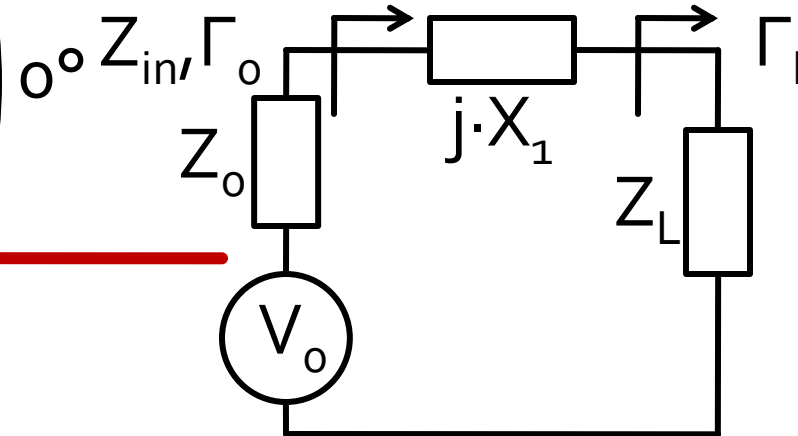
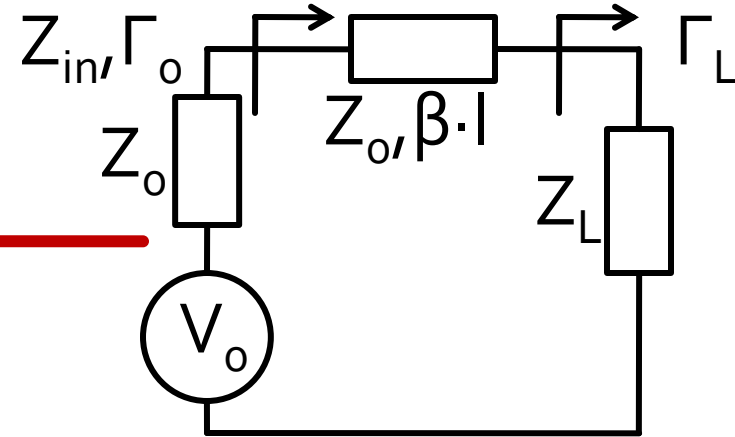
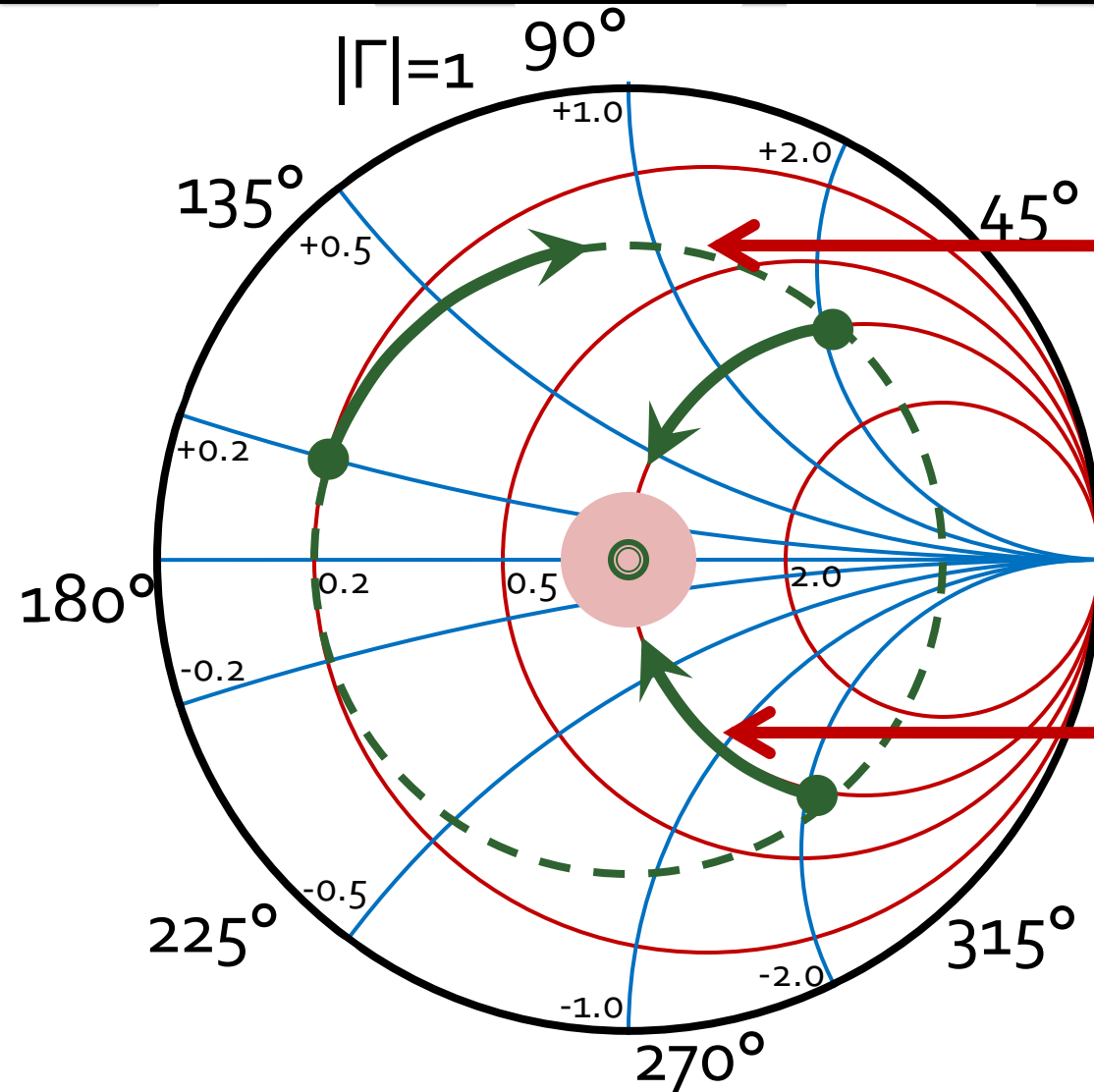


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$r_{in} = 1$$

Analytical solution, Γ

$$\cos(\varphi + 2\theta) = |\Gamma_s|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

$$\Gamma_s = 0.555 \angle -29.92^\circ$$

$$|\Gamma_s| = 0.555; \quad \varphi = -29.92^\circ \quad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

- **"+" solution**

$$(-29.92^\circ + 2\theta) = +56.28^\circ \quad \theta = 43.1^\circ \quad \text{Im } z_s = \frac{+2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\text{Im } z_s) = -36.8^\circ (+180^\circ) \rightarrow \theta_{ss} = 143.2^\circ$$

- **"-" solution**

$$(-29.92^\circ + 2\theta) = -56.28^\circ \quad \theta = -13.2^\circ (+180^\circ) \rightarrow \theta = 166.8^\circ$$

$$\text{Im } z_s = \frac{-2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = -1.335 \quad \theta_{ss} = -\cot^{-1}(\text{Im } z_s) = 36.8^\circ$$

Analytical solution, Γ

$$(\varphi + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \quad \theta = \begin{cases} 43.1^\circ \\ 166.8^\circ \end{cases} \quad \text{Im}[z_s(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8^\circ + 180^\circ = 143.2^\circ \\ +36.8^\circ \end{cases}$$

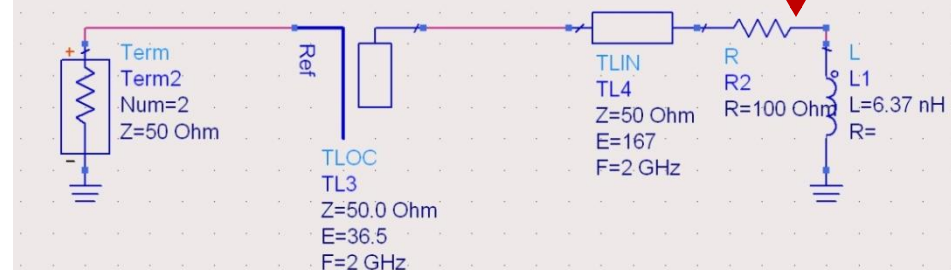
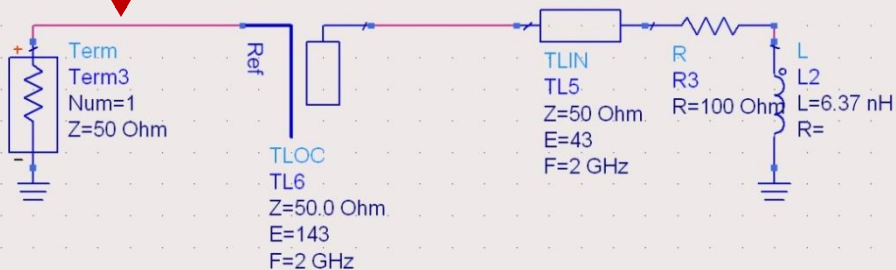
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

$$l_1 = \frac{43.1^\circ}{360^\circ} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2^\circ}{360^\circ} \cdot \lambda = 0.398 \cdot \lambda$$

$$l_1 = \frac{166.8^\circ}{360^\circ} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8^\circ}{360^\circ} \cdot \lambda = 0.102 \cdot \lambda$$

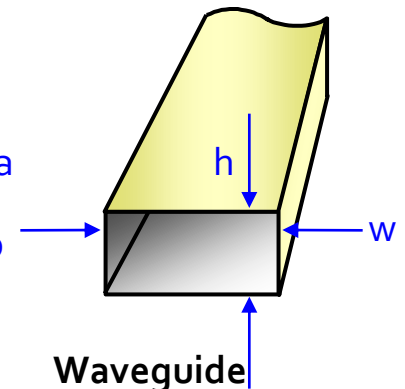
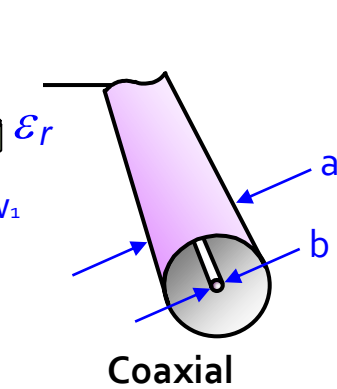
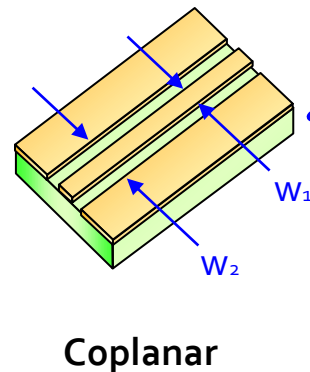
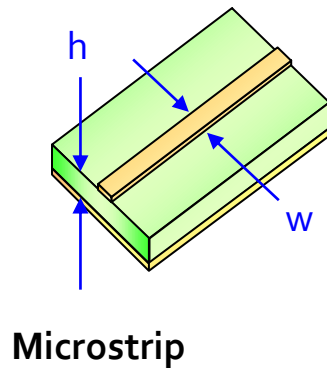


Single stub tuning

- We choose one of the 8 possible solutions (series/shunt, oc./sc.), taking into account:
 - physical dimensions (area occupied on chip/board)
 - sensitivity of the match on length error ($\Delta\Gamma/\Delta E$, $\Delta\Gamma/\Delta l$)
 - convenient frequency behavior (bandwidth)

Single stub tuning

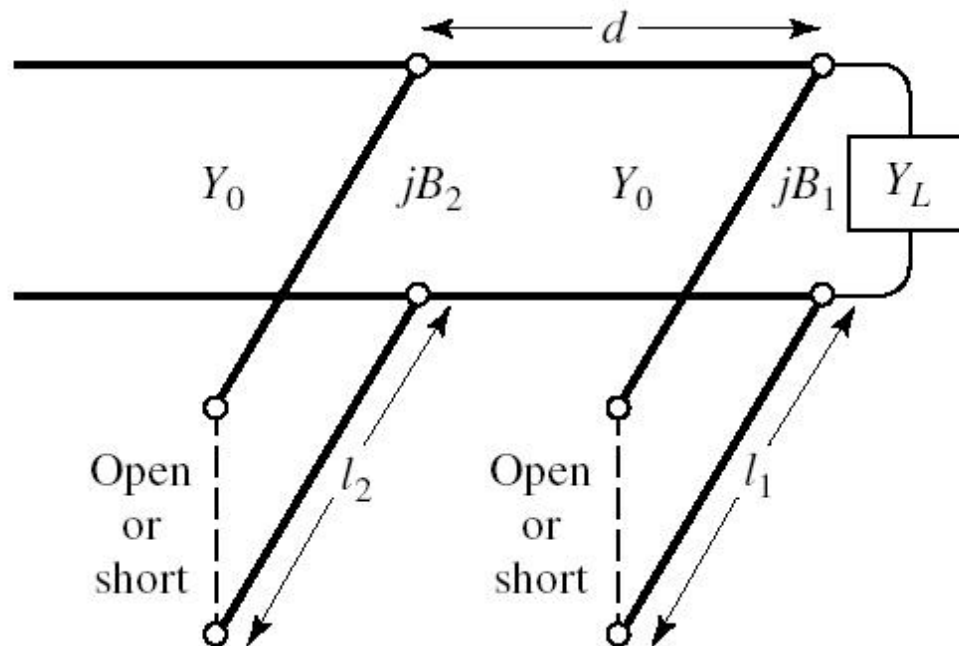
- We choose one of the 8 possible solutions (series/shunt, oc./sc.), taking into account :
 - physical realizability (in the line technology we use)



- Main disadvantage:
 - requires a variable length of line between the load and the stub

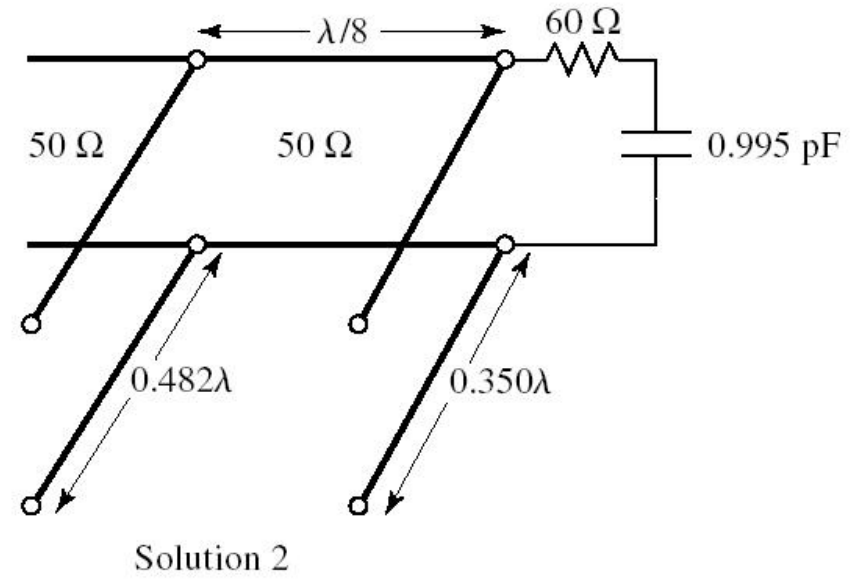
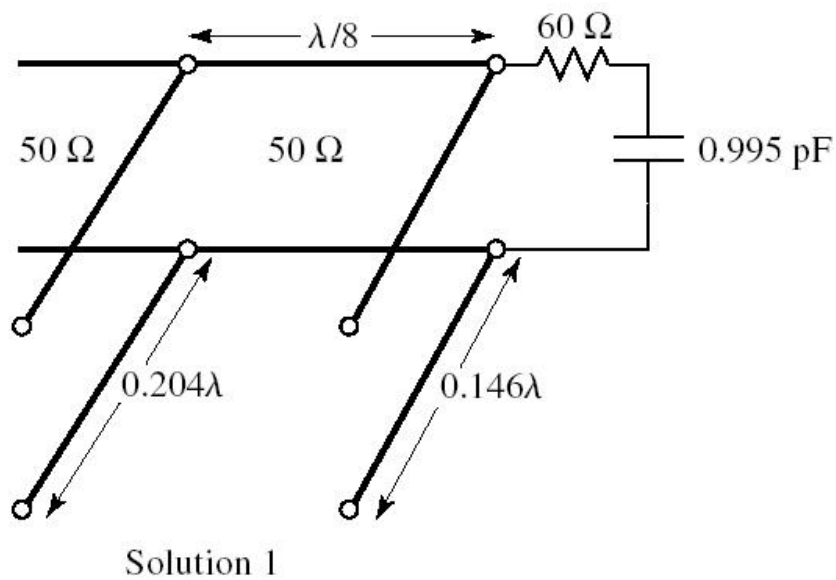
Double stub tuning

- Double stub tuning
- uses two tuning stubs in fixed positions (a fixed length of line between the stubs)



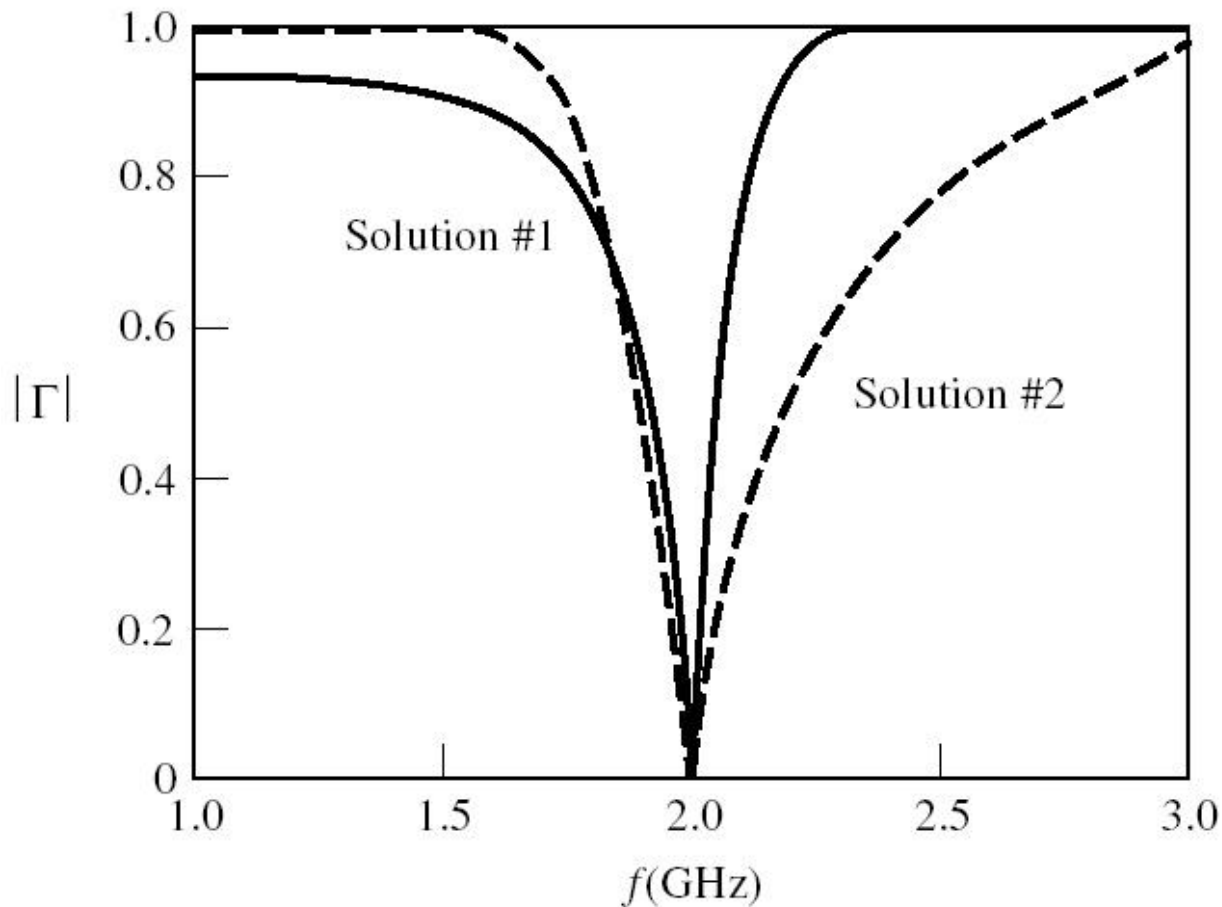
Double stub tuning

- same load: $60\ \Omega$ series with $0.995\ \text{pF}$ at $2\ \text{GHz}$
- two possible solutions



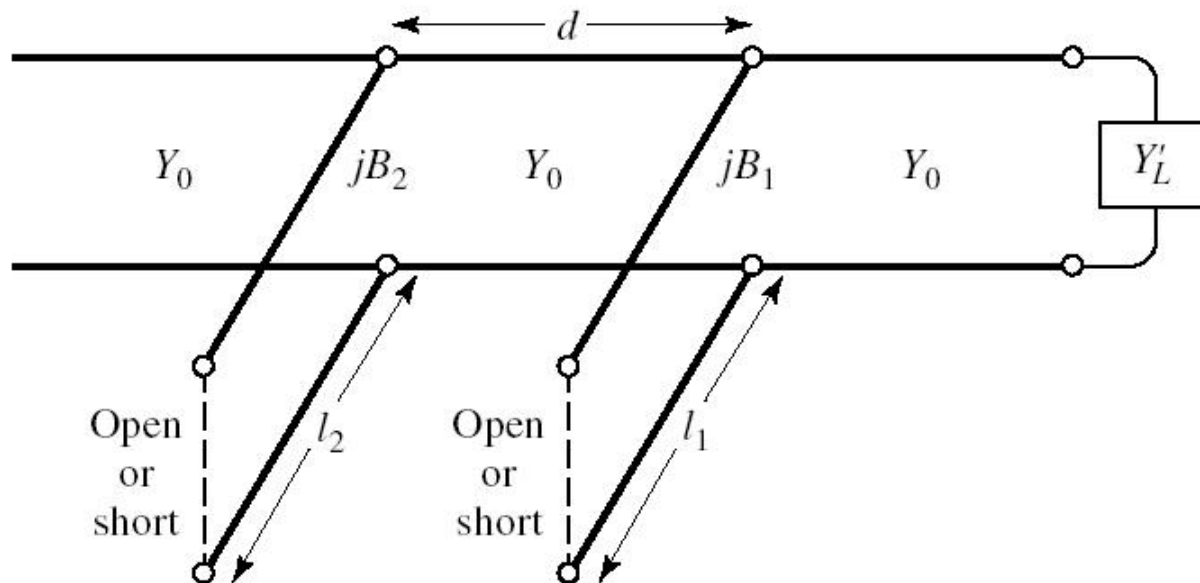
Double stub tuning

- two possible solutions

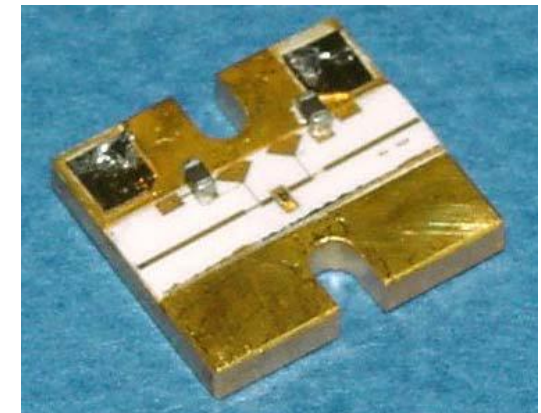
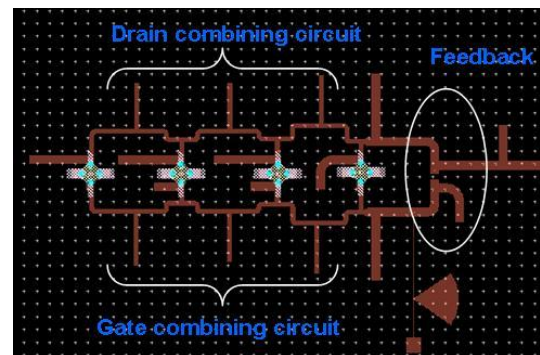
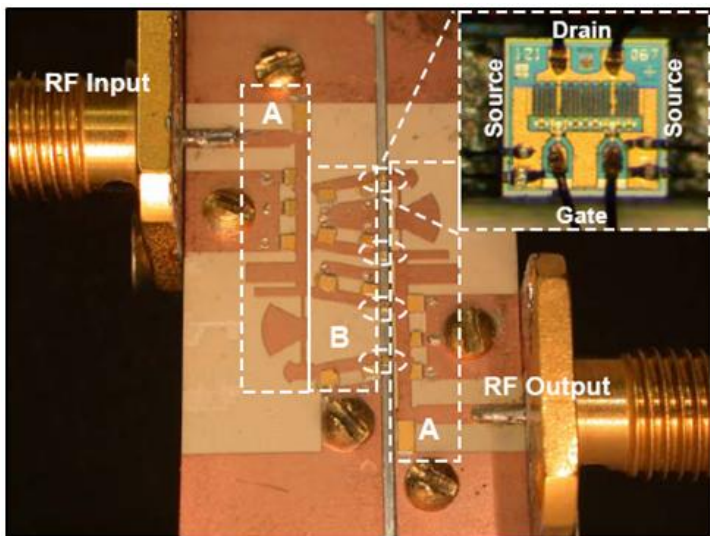
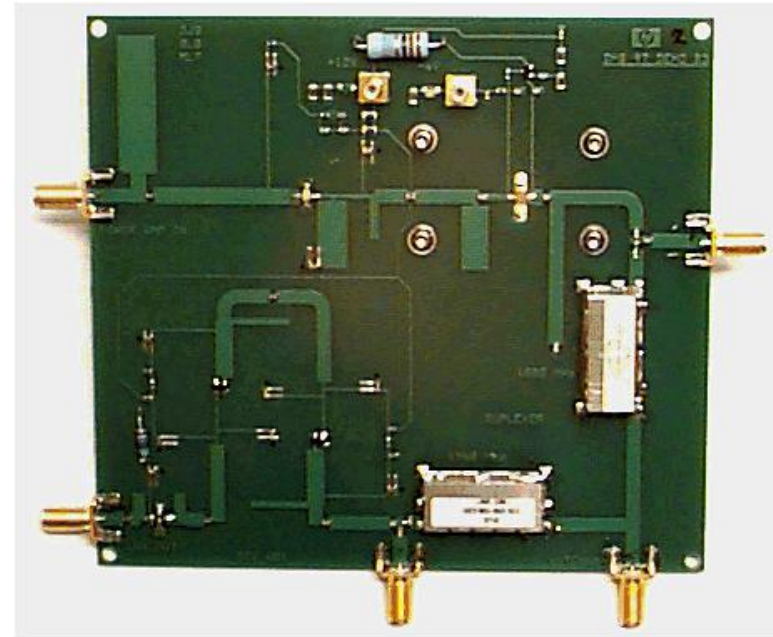
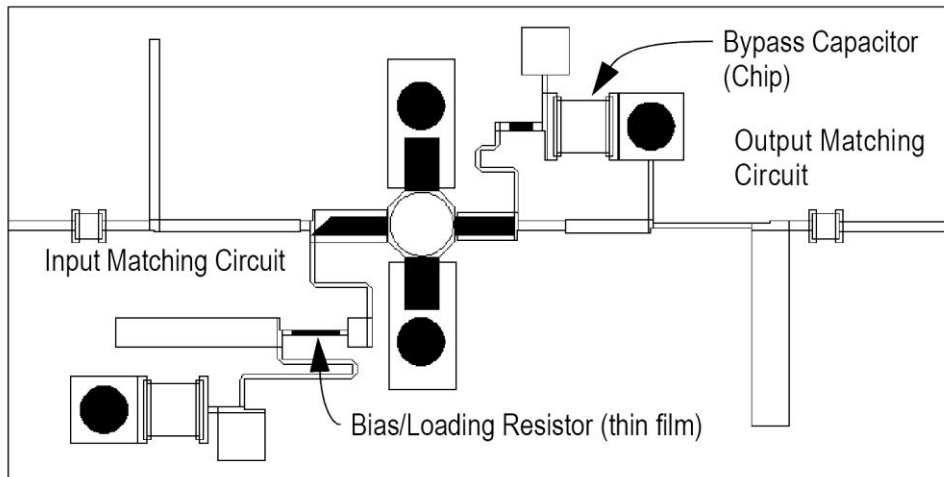


Double stub tuning

- Typically $d = \lambda/8$ or $d = 3\lambda/8$
- **Not possible** for every load
 - unless we can add a specific length of line between the load and the first stub

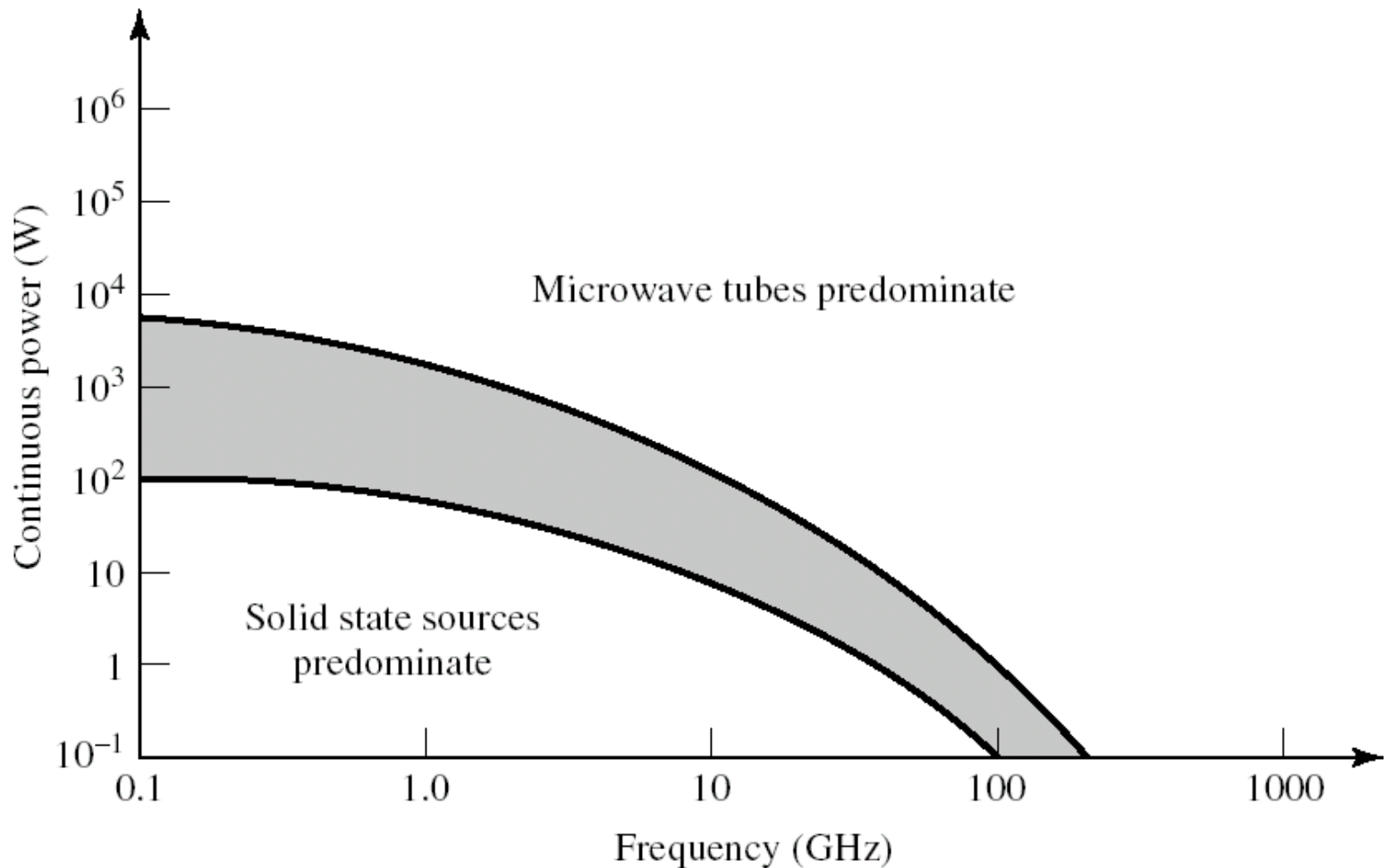


Impedance Matching with Stubs

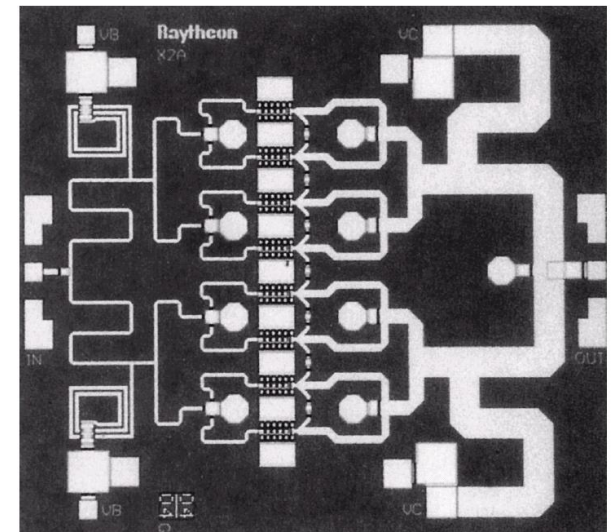
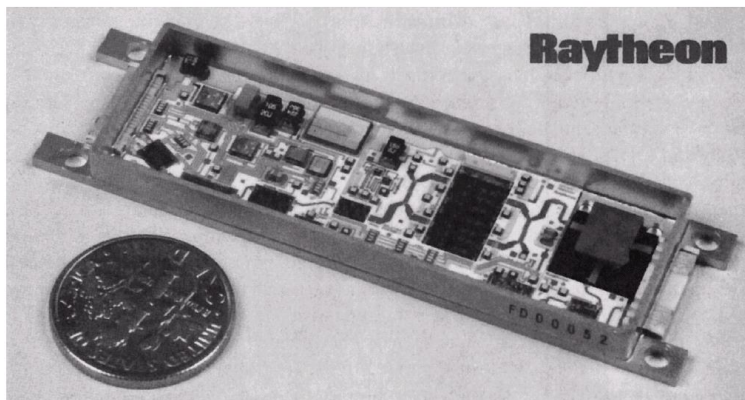
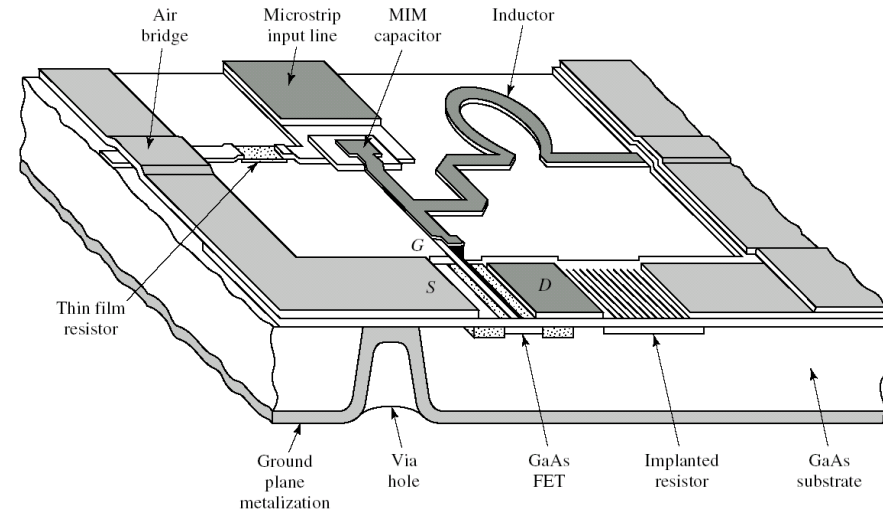
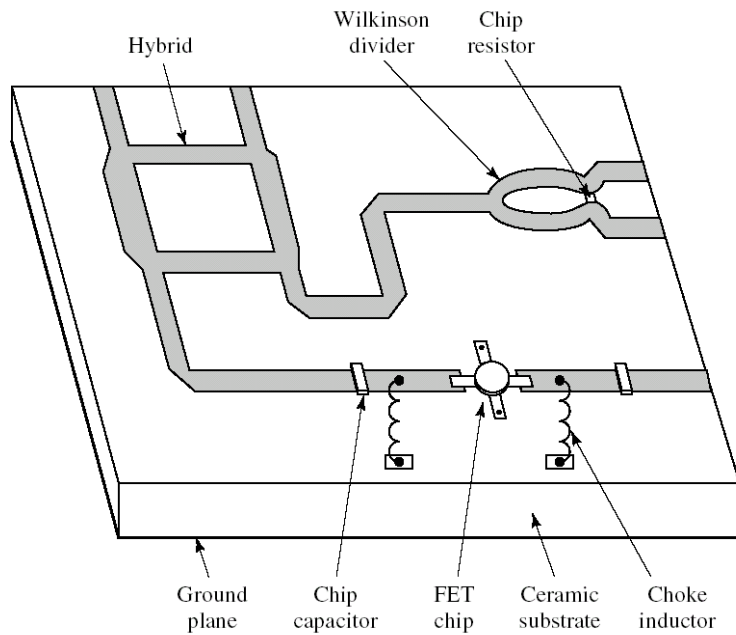


Microwave Amplifiers

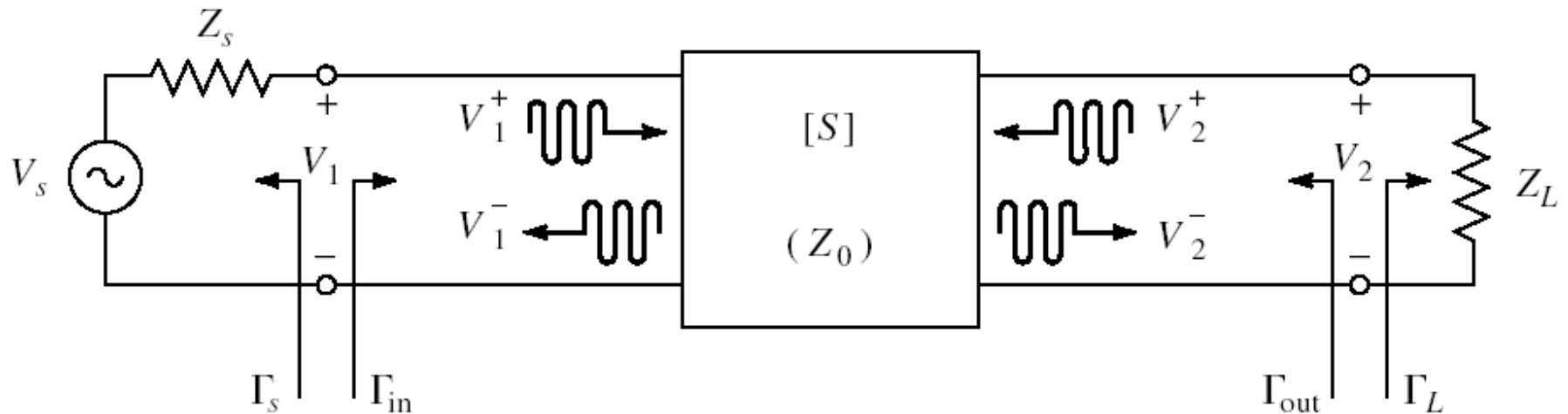
Microwave Amplifiers



Microwave Integrated Circuits



Amplifier as two-port



- Charaterized with S parameters
- normalized at Z_0 (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

Datasheets



NE46100 / NE46134

NPN MEDIUM POWER MICROWAVE TRANSISTOR

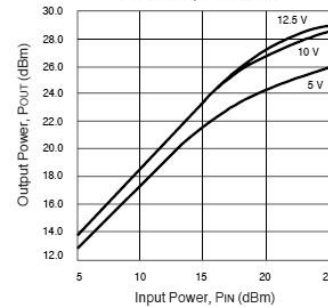
FEATURES

- HIGH DYNAMIC RANGE
- LOW IM DISTORTION: -40 dBc
- HIGH OUTPUT POWER : 27.5 dBm at TYP
- LOW NOISE: 1.5 dB TYP at 500 MHz
- LOW COST

DESCRIPTION

The NE461 series of NPN silicon epitaxial bipolar transistors is designed for medium power applications requiring high dynamic range. This device exhibits an outstanding combination of high gain and low intermodulation distortion, as well as low noise figure. The NE461 series offers excellent performance and reliability at low cost through titanium, platinum, gold metallization system and direct nitride passivation of the surface of the chip. Devices are available in a low cost surface mount package (SOT-89) as well as in chip form.

NE46134
TYPICAL OUTPUT POWER
vs. INPUT POWER
f = 1.0 GHz, I_c = 100 mA



ELECTRICAL CHARACTERISTICS (T_A = 25°C)

SYMBOLS	PARAMETERS AND CONDITIONS	UNITS	NE46100			NE46134 2SC4536 34		
			MIN	TYP	MAX	MIN	TYP	MAX
f _T	Gain Bandwidth Product at V _{CE} = 10 V, I _c = 100 mA	GHz		5.5			5.5	
N _{FMN}	Minimum Noise Figure ³ at V _{CE} = 10 V, I _c = 50 mA, 500 MHz V _{CE} = 10 V, I _c = 50 mA, 1 GHz	dB		1.5			1.5	
GL	Linear Gain, V _{CE} = 12.5 V, I _c = 100 mA, 2.0 GHz V _{CE} = 12.5 V, I _c = 100 mA, 1.0 GHz	dB		9.0			8.0	
IS _{21E1} ²	Insertion Power Gain at 10 V, 50 mA, f = 1.0 GHz	dB		10.0		5.5	7.0	
h _{FE}	DC Current Gain ² at V _{CE} = 10 V, I _c = 50 mA			40		40		200
I _{CB0}	Collector Cutoff Current at V _{CB} = 20 V, I _E = 0 mA	μA				5.0		5.0
I _{EB0}	Emitter Cutoff Current at V _{EB} = 2 V, I _C = 0 mA	μA				5.0		5.0
P _{1dB}	Output Power at 1 dB Compression, V _{CE} = 12.5 V, I _c = 100 mA, 2.0 GHz V _{CE} = 12.5 V, I _c = 100 mA, 1.0 GHz	dBm	27.0					27.5
IM ₃	Intermodulation Distortion, 10 V, 100 mA, F ₁ = 1.0 GHz, F ₂ = 0.99 GHz							

Datasheets

NE46100

VCE = 5 V, Ic = 50 mA

FREQUENCY (MHz)	S11		S21		S12		S22		K	MAG ² (dB)
	MAG	ANG	MAG	ANG	MAG	ANG	MAG	ANG		
100	0.778	-137	26.776	114	0.028	30	0.555	-102	0.16	29.8
200	0.815	-159	14.407	100	0.035	29	0.434	-135	0.36	26.2
500	0.826	-177	5.855	84	0.040	38	0.400	-162	0.75	21.7
800	0.827	176	3.682	76	0.052	43	0.402	-169	0.91	18.5
1000	0.826	173	2.963	71	0.058	47	0.405	-172	1.02	16.3
1200	0.825	170	2.441	66	0.064	47	0.412	-174	1.08	14.0
1400	0.820	167	2.111	61	0.069	47	0.413	-176	1.17	12.4
1600	0.828	165	1.863	57	0.078	54	0.426	-177	1.15	11.4
1800	0.827	162	1.671	53	0.087	50	0.432	-178	1.14	10.6
2000	0.828	159	1.484	49	0.093	50	0.431	-180	1.17	9.5
2500	0.822	153	1.218	39	0.11	48	0.462	177	1.18	7.8
3000	0.818	148	1.010	30	0.135	46	0.490	174	1.16	6.3
3500	0.824	142	0.876	21	0.147	44	0.507	170	1.16	5.3
4000	0.812	137	0.762	13	0.168	38	0.535	167	1.14	4.3

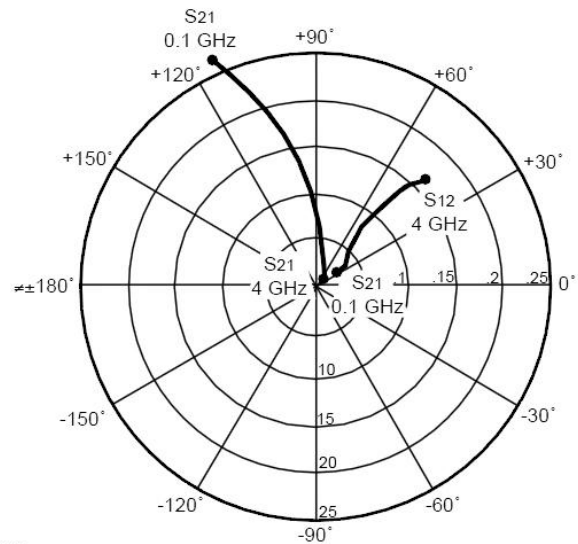
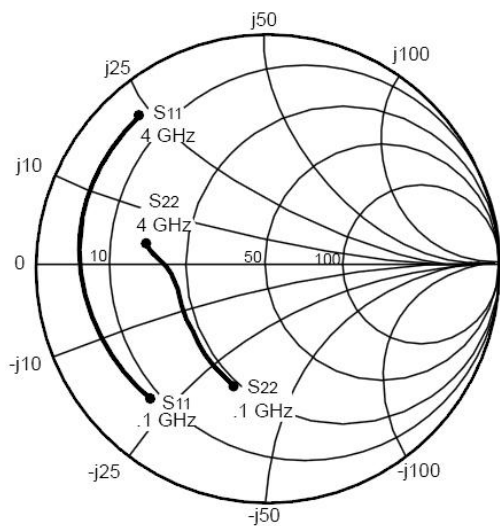
VCE = 5 V, Ic = 100 mA

100	0.778	-144	27.669	111	0.027	35	0.523	-114	0.27	30.2
200	0.820	-164	14.559	97	0.029	29	0.445	-144	0.42	27.0
500	0.832	-179	5.885	84	0.035	38	0.435	-166	0.81	22.2
800	0.833	175	3.691	76	0.048	45	0.435	-173	0.95	18.8
1000	0.831	172	2.980	71	0.056	51	0.437	-176	1.05	16.0
1200	0.836	169	2.464	67	0.061	52	0.432	-178	1.11	14.0
1400	0.829	166	2.121	61	0.072	53	0.447	-180	1.12	12.6
1600	0.831	164	1.867	58	0.080	54	0.445	179	1.14	11.4

Datasheets

NE46100, NE46134

TYPICAL COMMON EMITTER SCATTERING PARAMETERS¹ ($T_A = 25^\circ\text{C}$)



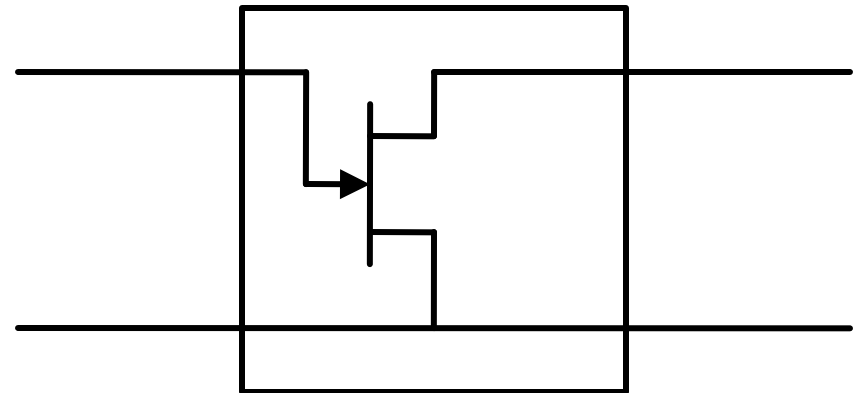
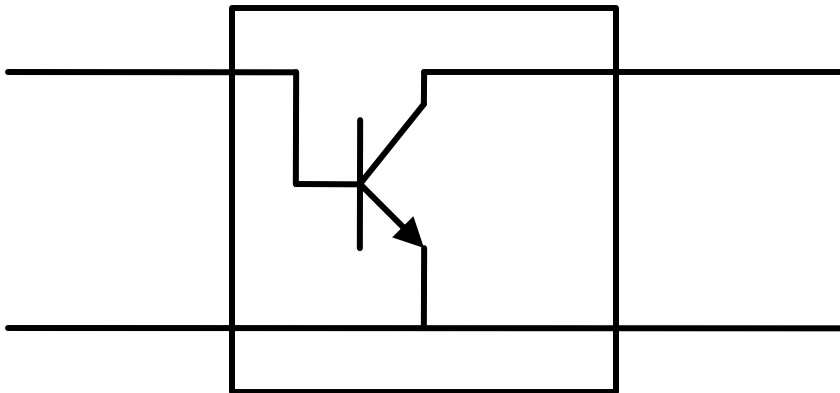
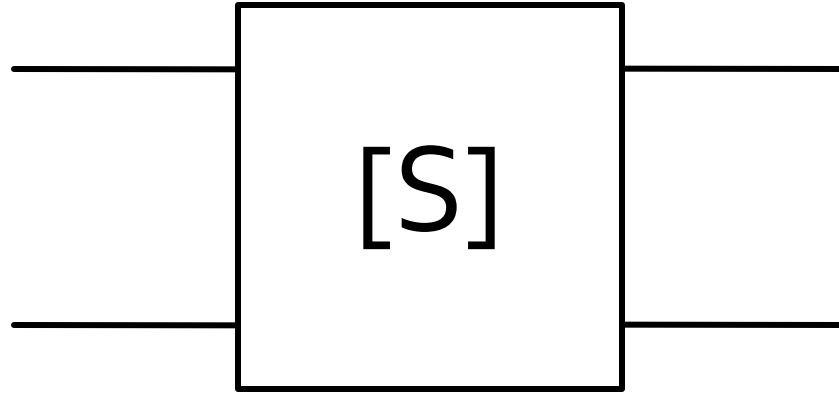
Coordinates in Ohms
Frequency in GHz
 $V_{CE} = 5\text{ V}$, $I_c = 50\text{ mA}$

S2P - Touchstone

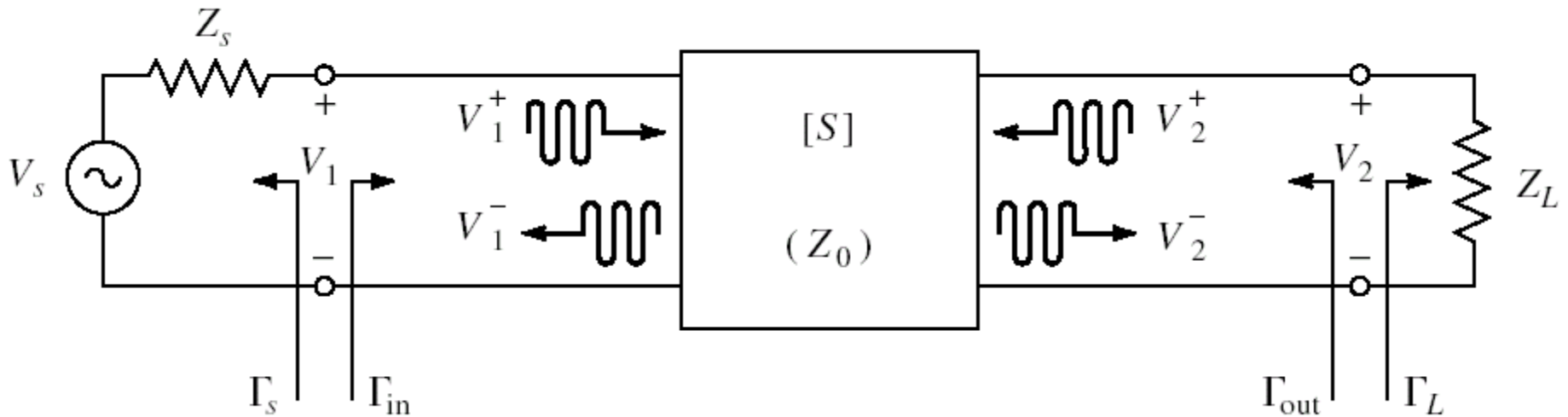
- Touchstone file format (*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V   ID = 15 mA
# GHz S MA R 50
! f      S11      S21      S12      S22
! GHz  MAG ANG  MAG ANG  MAG ANG  MAG ANG
1.000 0.9800 -18.0 2.230 157.0 0.0240 74.0 0.6900 -15.0
2.000 0.9500 -39.0 2.220 136.0 0.0450 57.0 0.6600 -30.0
3.000 0.8900 -64.0 2.210 110.0 0.0680 40.0 0.6100 -45.0
4.000 0.8200 -89.0 2.230 86.0 0.0850 23.0 0.5600 -62.0
5.000 0.7400 -115.0 2.190 61.0 0.0990 7.0 0.4900 -80.0
6.000 0.6500 -142.0 2.110 36.0 0.1070 -10.0 0.4100 -98.0
!
! f      Fmin  Gammaopt  rn/50
! GHz    dB   MAG ANG  -
2.000    1.00  0.72  27  0.84
4.000    1.40  0.64  61  0.58
```

S parameters for transistors



Amplifier as two-port

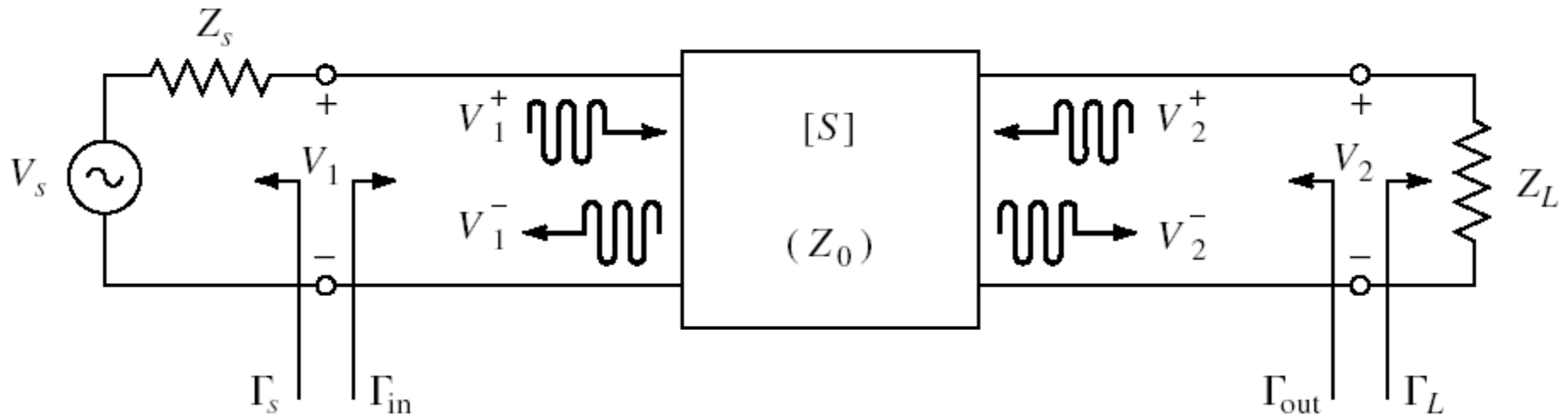


$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \quad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$\Gamma_L = \frac{V_2^+}{V_2^-} \quad V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

Amplifier as two-port



$$V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

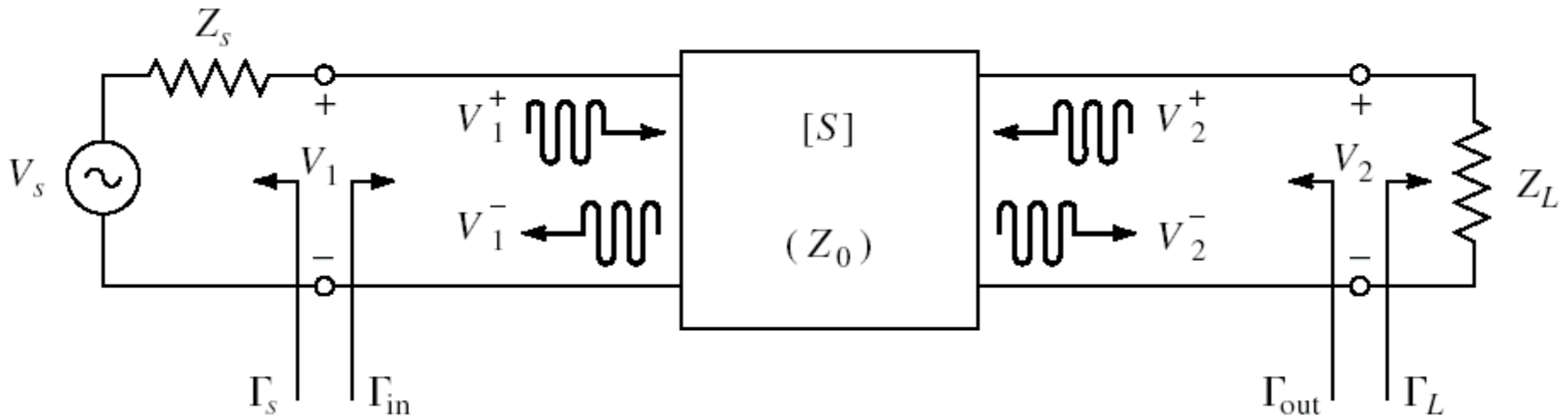
$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

■ similarly

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Amplifier as two-port



$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_s}{1 - S_{11} \cdot \Gamma_s}$$

Signal power

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$V_1 = \frac{V_S \cdot Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+ \cdot (1 + \Gamma_{in})$$

$$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \cdot \Gamma_{in})}$$

■ C3 $P_{in} = \frac{1}{2 \cdot Z_0} \cdot |V_1^+|^2 \cdot (1 - |\Gamma_{in}|^2)$

$$P_L = \frac{1}{2 \cdot Z_0} \cdot |V_2^-|^2 \cdot (1 - |\Gamma_L|^2)$$

$$P_{in} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2} (1 - |\Gamma_{in}|^2)$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = \frac{S_{21} \cdot V_1^+}{1 - S_{22} \cdot \Gamma_L}$$

$$P_L = \frac{|V_1^+|^2}{2 \cdot Z_0} \cdot \frac{|S_{21}|^2}{|1 - S_{22} \cdot \Gamma_L|^2} (1 - |\Gamma_L|^2)$$

$$P_L = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{|1 - S_{22} \cdot \Gamma_L|^2} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2}$$

Signal power

- Signal power

$$P_{in} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2} \left(1 - |\Gamma_{in}|^2\right)$$

$$P_L = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{|1 - S_{22} \cdot \Gamma_L|^2} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2}$$

- Power available from the source

$$P_{av S} = P_{in}|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{(1 - |\Gamma_S|^2)}$$

- Power available on the load (from the network)

$$P_{av L} = P_L|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot |1 - \Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2 \cdot (1 - |\Gamma_{out}|^2)}$$

Two-Port Power Gains

- Power Gain

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2) \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$P_{in} = P_{in}(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

$$P_L = P_L(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

- The **actual** power gain **introduced** by the amplifier is less important because a higher gain may be accompanied by a **decrease** in input power (power actually drained from the source)
- We prefer to characterize the amplifier effect looking to the **power actually delivered to the load** in relation to the power **available from the source** (which is a constant)

Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av L}}{P_{av S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2)}{|1 - S_{22} \cdot \Gamma_L|^2 \cdot (1 - |\Gamma_{out}|^2)}$$

- **Transducer** power gain

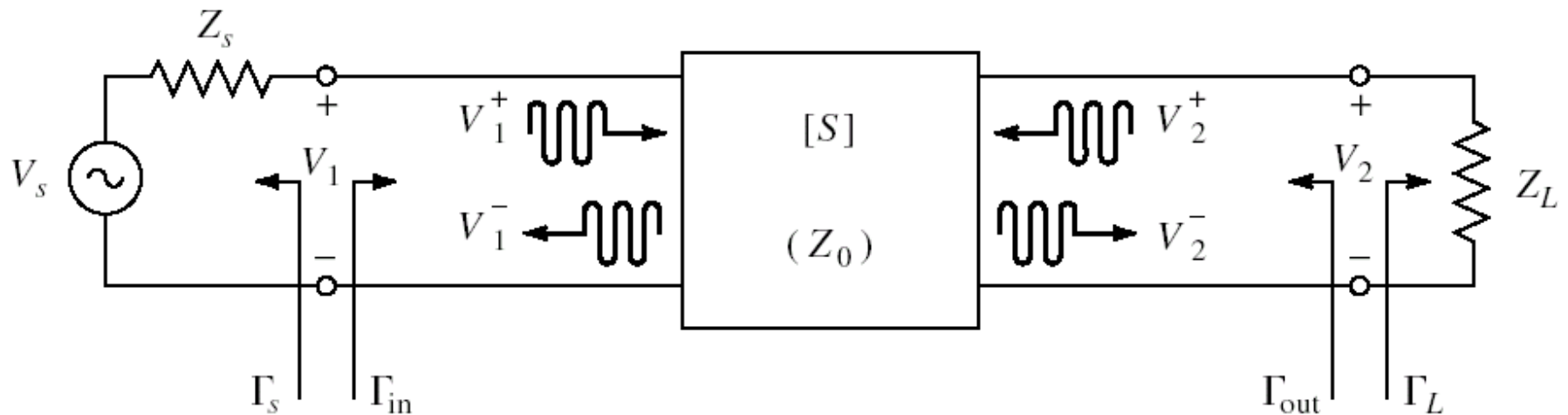
$$G_T = \frac{P_L}{P_{av S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2) \cdot (1 - |\Gamma_L|^2)}{|1 - \Gamma_S \cdot \Gamma_{in}|^2 \cdot |1 - S_{22} \cdot \Gamma_L|^2} \quad \Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2} \quad S_{12} \cong 0 \quad \Gamma_{in} = S_{11}$$

Input and output can be treated independently

Amplifier as two-port

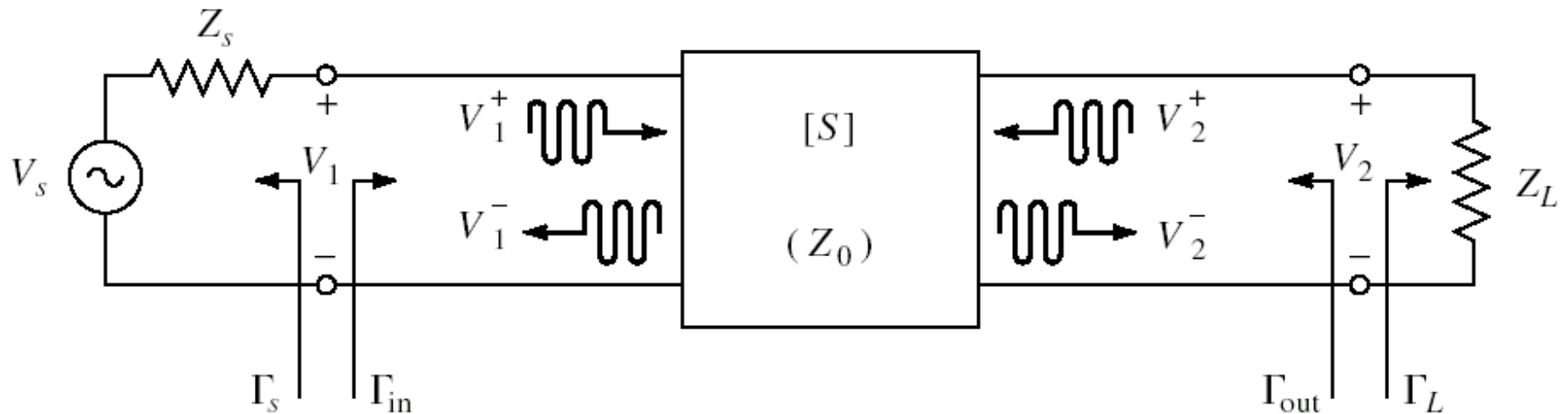


- For an amplifier two-port we are interested in:
 - stability
 - power gain
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

Stability

Microwave Amplifiers

Amplifier as two-port



- For an amplifier two-port we are interested in:
 - **stability**
 - power gain
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

Stability

- $$L5 \quad \Gamma = \Gamma_r + j \cdot \Gamma_i \quad r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$Z_{in} \quad \Gamma_{in} = \Gamma_r + j \cdot \Gamma_i$$

- instability

$$\text{Re}\{Z_{in}\} < 0 \iff 1 - \Gamma_r^2 - \Gamma_i^2 < 0 \quad |\Gamma_{in}| > 1$$

- stability, Z_{in}

- conditions to be met by Γ_L to achieve (input) stability

$$|\Gamma_{in}| < 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- similarly Z_{out}

- conditions to be met by Γ_S to achieve (output) stability

Stability

$$|\Gamma_{in}| < 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- We can calculate conditions to be met by Γ_L to achieve stability

$$|\Gamma_{out}| < 1 \quad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| < 1$$

- We can calculate conditions to be met by Γ_S to achieve stability

Stability

$$|\Gamma_{in}| < 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- The limit between stability/instability

$$|\Gamma_{in}| = 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| = 1$$

$$|S_{11} \cdot (1 - S_{22} \cdot \Gamma_L) + S_{12} \cdot S_{21} \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

- determinant of the S matrix $\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$

$$|S_{11} - \Delta \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

$$|S_{11} - \Delta \cdot \Gamma_L|^2 = |1 - S_{22} \cdot \Gamma_L|^2$$

Stability

$$|S_{11} - \Delta \cdot \Gamma_L|^2 = |1 - S_{22} \cdot \Gamma_L|^2$$

$$a \cdot a^* = |a| \cdot e^{j\theta} \cdot |a| \cdot e^{-j\theta} = |a|^2$$

$$|a+b|^2 = (a+b) \cdot (a+b)^* = (a+b) \cdot (a^* + b^*) = \underbrace{|a|^2} + \underbrace{|b|^2} + \underbrace{a^* \cdot b + a \cdot b^*}$$

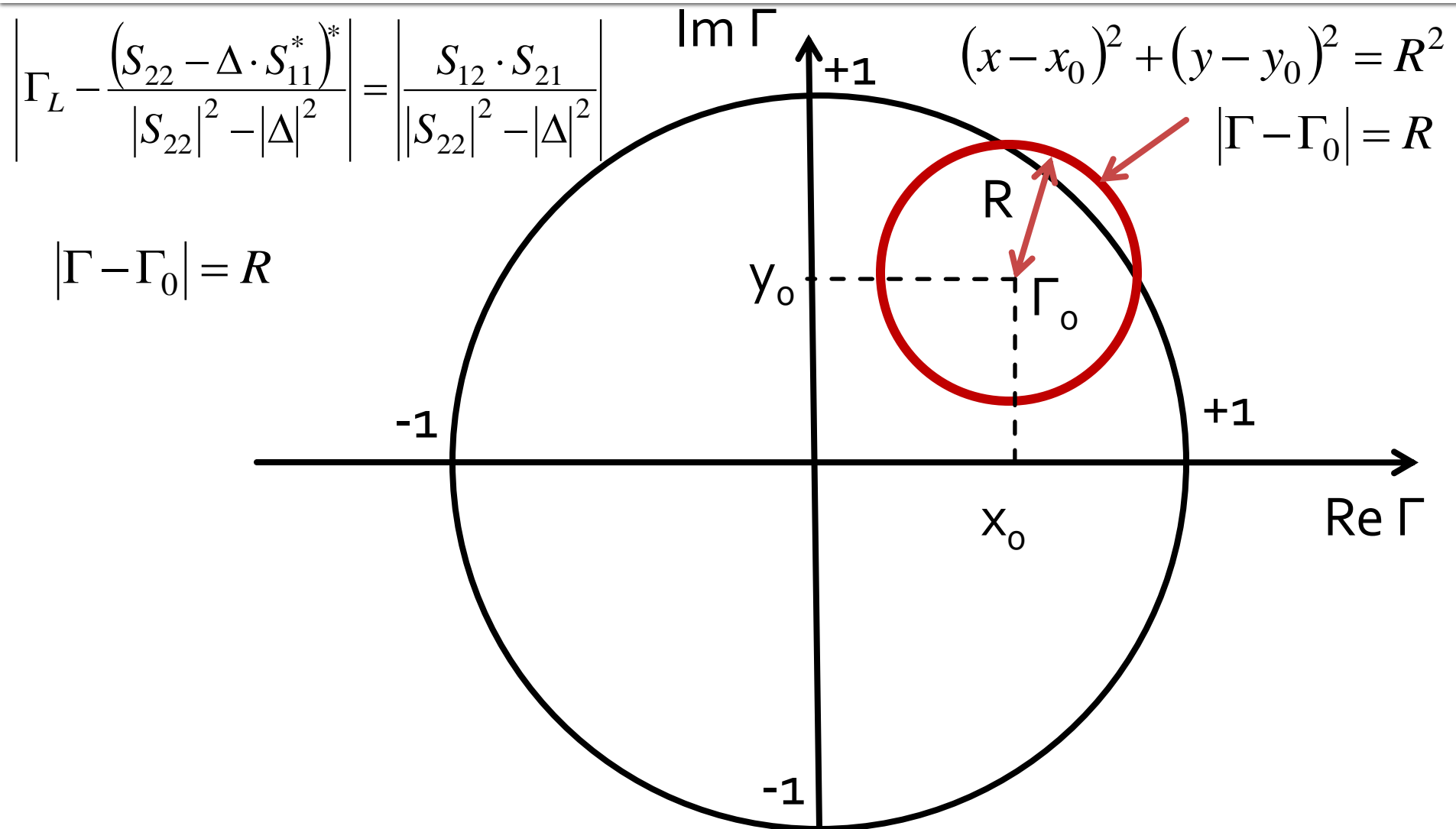
$$|S_{11}|^2 + |\Delta|^2 \cdot |\Gamma_L|^2 - (\Delta \cdot \Gamma_L \cdot S_{11}^* + \Delta^* \cdot \Gamma_L^* \cdot S_{11}) = 1 + |S_{22}|^2 \cdot |\Gamma_L|^2 - (S_{22}^* \cdot \Gamma_L^* + S_{22} \cdot \Gamma_L)$$

$$(|S_{22}|^2 - |\Delta|^2) \cdot \Gamma_L \cdot \Gamma_L^* - (S_{22} - \Delta \cdot S_{11}^*) \cdot \Gamma_L - (S_{22}^* - \Delta^* \cdot S_{11}) \cdot \Gamma_L^* = |S_{11}|^2 - 1$$

$$\Gamma_L \cdot \Gamma_L^* - \frac{(S_{22} - \Delta \cdot S_{11}^*) \cdot \Gamma_L + (S_{22}^* - \Delta^* \cdot S_{11}) \cdot \Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2} + \frac{|S_{22} - \Delta \cdot S_{11}^*|^2}{(|S_{22}|^2 - |\Delta|^2)^2}$$

$$\left| \Gamma_L - \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right|^2 = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2} + \frac{|S_{22} - \Delta \cdot S_{11}^*|^2}{(|S_{22}|^2 - |\Delta|^2)^2}$$

Stability



Output stability circle (CSOUT)

$$\left| \Gamma_L - \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \quad |\Gamma_L - C_L| = R_L$$

- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** (Γ_L)

$$C_L = \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad R_L = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{22}|^2 - |\Delta|^2 \right|}$$

Input stability circle (CSIN)

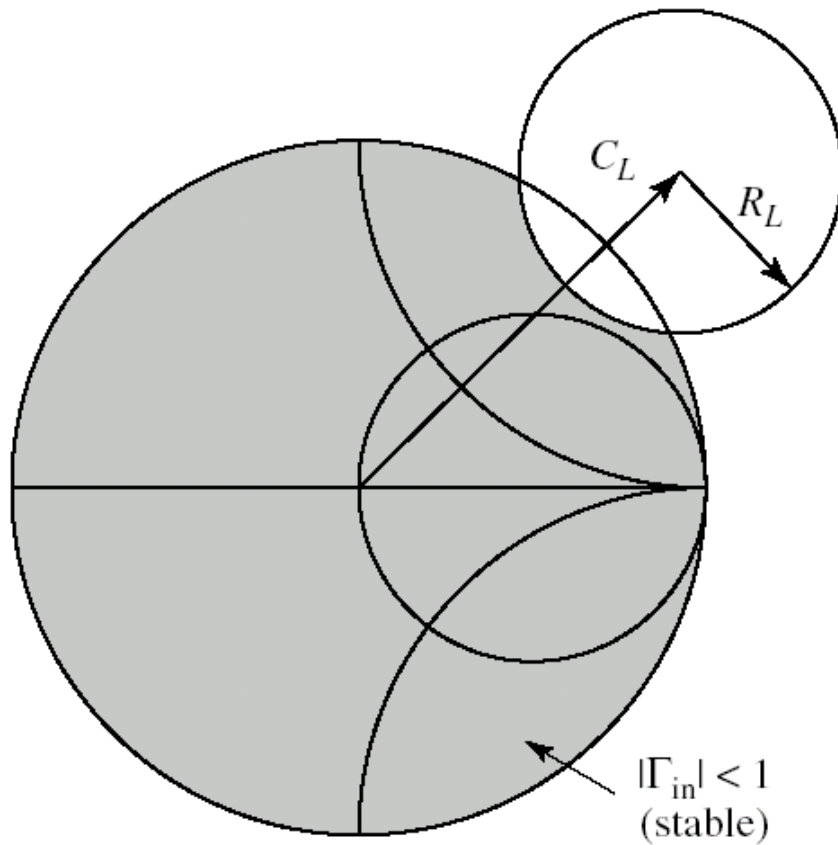
- Similarly $|\Gamma_{out}|=1$ $\left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| = 1$
- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_S for the **limit between stability and instability** ($|\Gamma_{out}| = 1$)
- This circle is the **input stability circle** (Γ_S)

$$C_S = \frac{(S_{11} - \Delta \cdot S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \quad R_S = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{11}|^2 - |\Delta|^2 \right|}$$

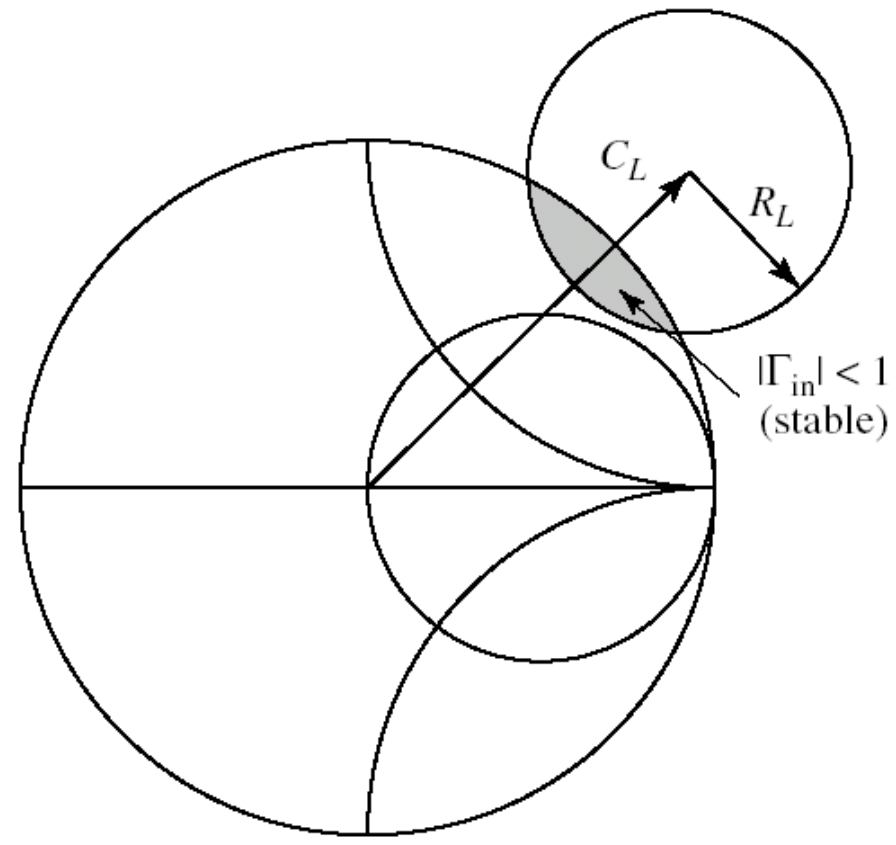
Output stability circle (CSOUT)

- The **output stability circle** represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- The circle divides the complex planes in two areas, the **inside** and the **outside** of the circle
- The two areas will represent the locus of Γ_L for stability ($|\Gamma_{in}| < 1$) / instability ($|\Gamma_{in}| > 1$)

Output stability circle (CSOUT)



(a)



(b)

- Two cases possible: (a) stable outside/ (b) stable inside

Output stability circle (CSOUT)

- Identification of the stability / instability regions
 - Center of the Smith Chart in Γ_L complex plane correspond to $\Gamma_L = 0$
 - Input reflection coefficient

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \quad \Gamma_{in}|_{\Gamma_L=0} = S_{11} \quad |\Gamma_{in}|_{\Gamma_L=0} = |S_{11}|$$

- A decision can be made based on $|S_{11}|$ value the position of the center of the Smith chart (origin of the complex plane) relative to the circle

Identification of the stability / instability regions

- Output stability circle
 - $|S_{11}| < 1 \rightarrow$ the center of the Smith chart on which Γ_L is represented is a **stable point**, so it's placed in the stability region (most often situation)
 - $|S_{11}| > 1 \rightarrow$ the center of the Smith chart on which Γ_L is represented is a **unstable point**, so it's placed in the instability region
- Input stability circle
 - $|S_{22}| < 1 \rightarrow$ the center of the Smith chart on which Γ_S is represented is a **stable point**, so it's placed in the stability region (most often situation)
 - $|S_{22}| > 1 \rightarrow$ the center of the Smith chart on which Γ_S is represented is a **unstable point**, so it's placed in the instability region

Example

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @5GHz
 - $S_{11} = 0.64 \angle 139^\circ$
 - $S_{12} = 0.119 \angle -21^\circ$
 - $S_{21} = 3.165 \angle 16^\circ$
 - $S_{22} = 0.22 \angle 146^\circ$

```
IATF-34143
IS-PARAMETERS at Vds=3V Id=20mA. LAST UPDATED 01-29-99

# ghz s ma r 50

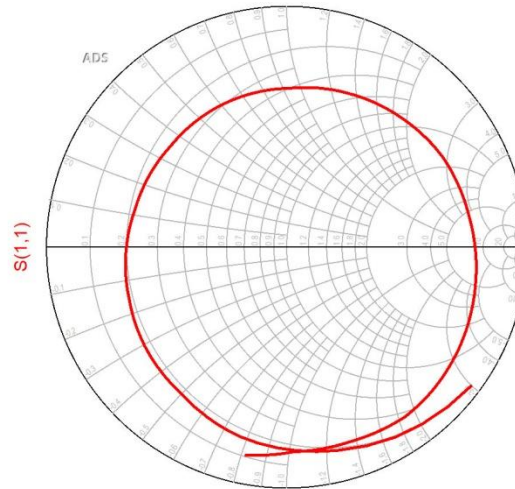
2.0 0.75 -126 6.306 90 0.088 23 0.26 -120
2.5 0.72 -145 5.438 75 0.095 15 0.25 -140
3.0 0.69 -162 4.762 62 0.102 7 0.23 -156
4.0 0.65 166 3.806 38 0.111 -8 0.22 174
5.0 0.64 139 3.165 16 0.119 -21 0.22 146
6.0 0.65 114 2.706 -5 0.125 -35 0.23 118
7.0 0.66 89 2.326 -27 0.129 -49 0.25 91
8.0 0.69 67 2.017 -47 0.133 -62 0.29 67
9.0 0.72 48 1.758 -66 0.135 -75 0.34 46

!FREQ Fopt GAMMA OPT RN/Zo
!GHZ dB MAG ANG -

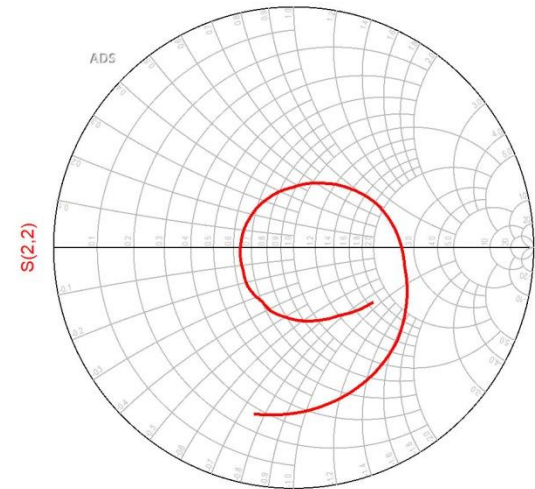
2.0 0.19 0.71 66 0.09
2.5 0.23 0.65 83 0.07
3.0 0.29 0.59 102 0.06
4.0 0.42 0.51 138 0.03
5.0 0.54 0.45 174 0.03
6.0 0.67 0.42 -151 0.05
7.0 0.79 0.42 -118 0.10
8.0 0.92 0.45 -88 0.18
9.0 1.04 0.51 -63 0.30
10.0 1.16 0.61 -43 0.46
```


Example

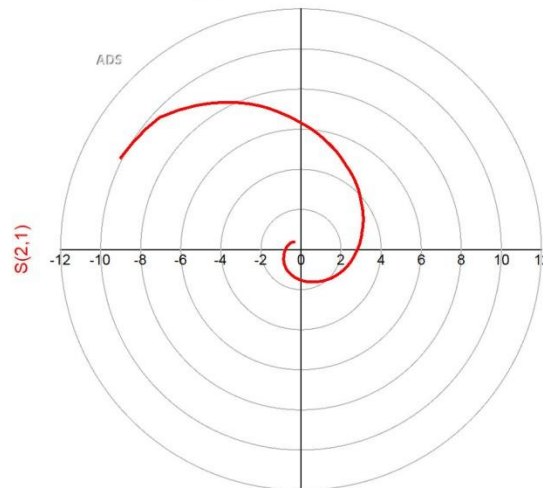
- ATF-34143
- at
 - $V_{ds}=3V$
 - $I_d=20mA$.



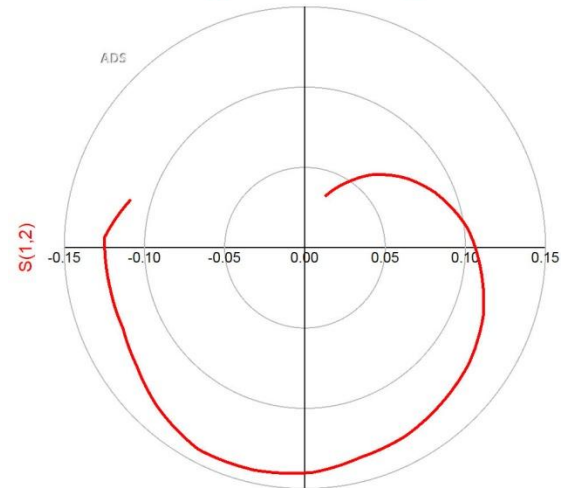
freq (500.0MHz to 18.00GHz)



freq (500.0MHz to 18.00GHz)



freq (500.0MHz to 18.00GHz)



freq (500.0MHz to 18.00GHz)

Solution + region identification

- S parameters

- $S_{11} = -0.483 + 0.42 \cdot j$

- $S_{12} = 0.111 - 0.043 \cdot j$

- $S_{21} = 3.042 + 0.872 \cdot j$

- $S_{22} = -0.182 + 0.123 \cdot j$

$$C_L = \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 3.931 - 0.897 \cdot j$$

$$|C_L| = 4.032$$

$$R_L = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{22}|^2 - |\Delta|^2 \right|} = 4.891$$

- $|S_{22}| < 1$

- $|C_L| < R_L, o \in \text{CSOUT}$

- The center of the Smith chart is placed inside the output stability circle ($o \in \text{CSOUT}$) and is a stable point

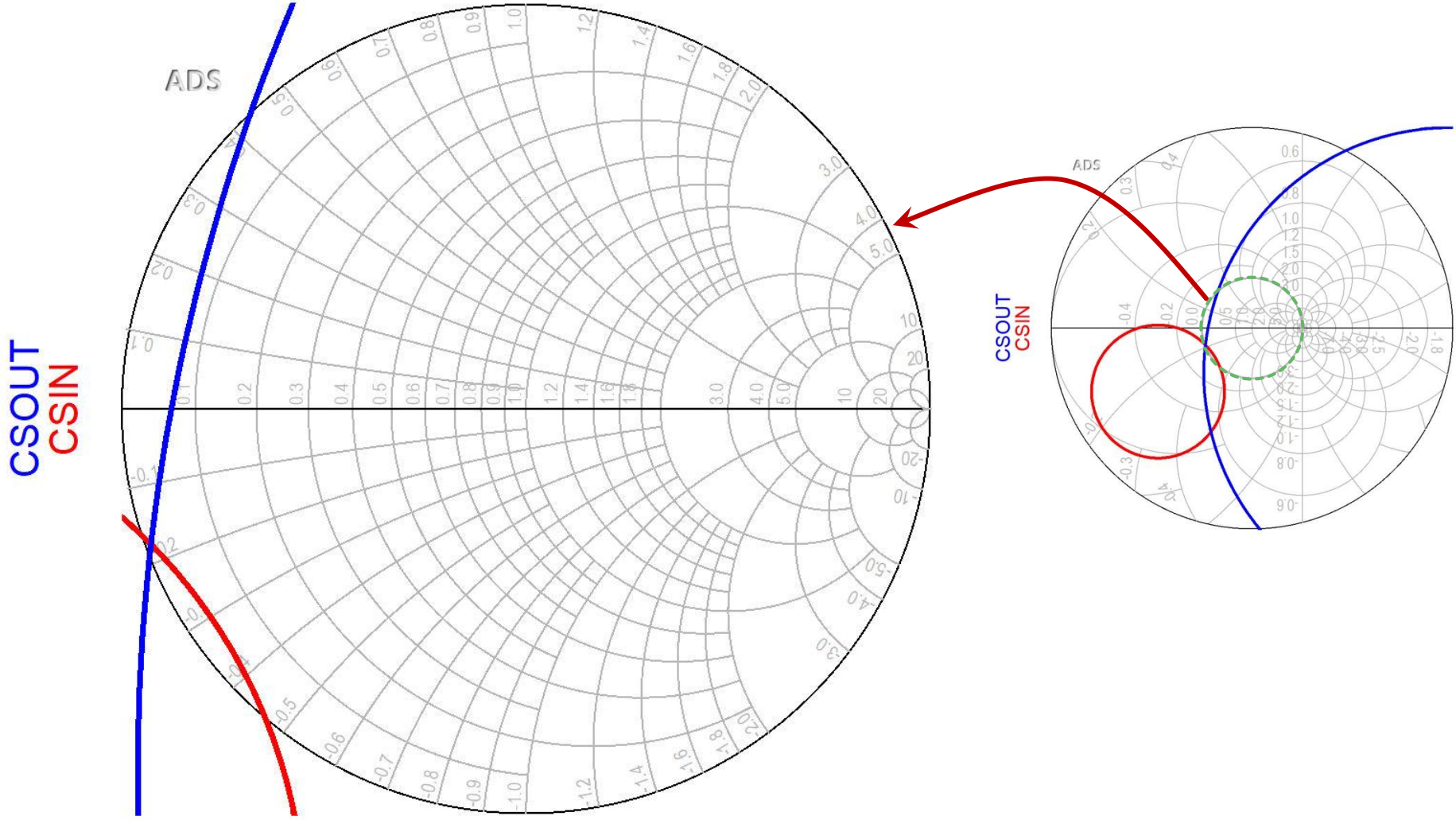
- the inside of the output stability circle – stability region

- the outside of the output stability circle – instability region

Solution + region identification

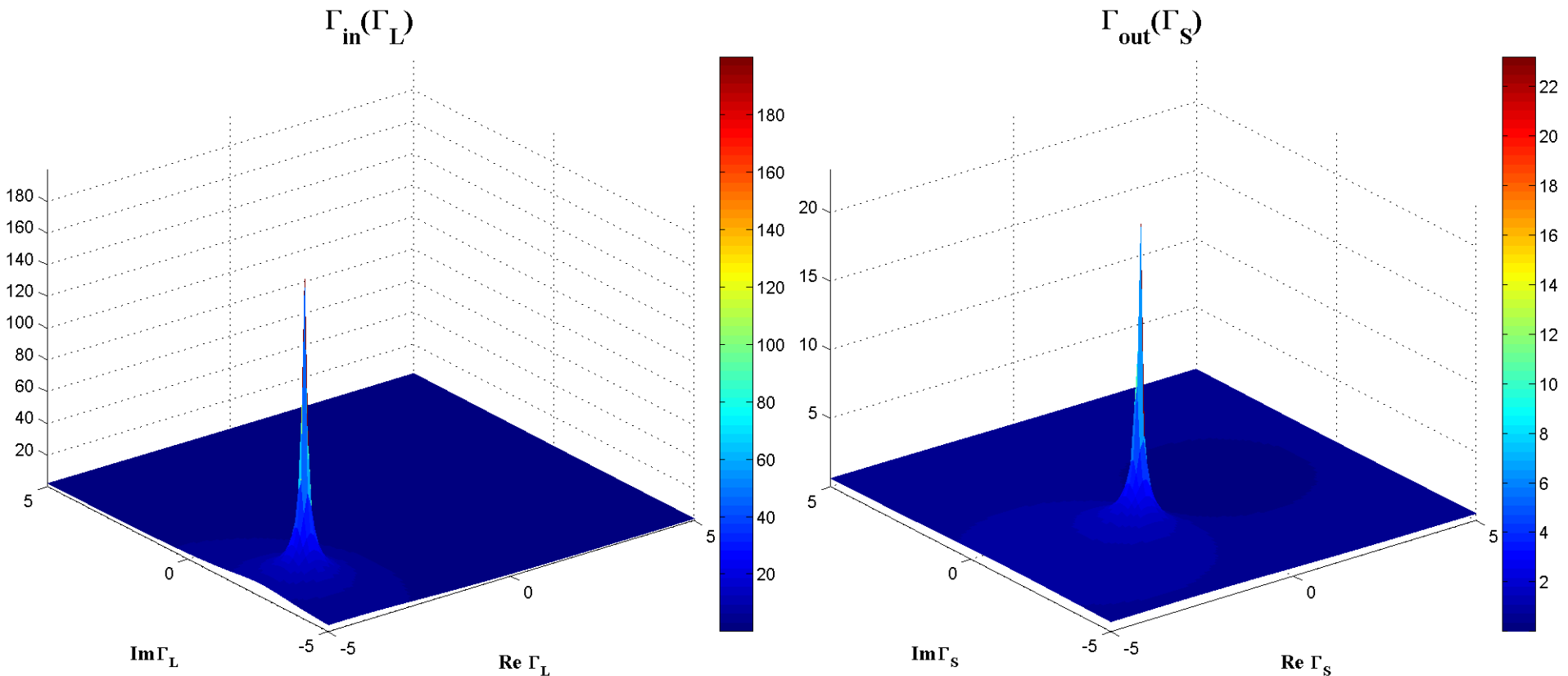
- S parameters
 - $S_{11} = -0.483 + 0.42 \cdot j$
 - $S_{12} = 0.111 - 0.043 \cdot j$
 - $S_{21} = 3.042 + 0.872 \cdot j$
 - $S_{22} = -0.182 + 0.123 \cdot j$
 - $|S_{11}| < 1$
 - $|C_S| > R_S, 0 \notin \text{CSIN}$
 - The center of the Smith chart is placed outside the input stability circle ($0 \notin \text{CSIN}$) and is a stable point
 - the outside of the input stability circle – stability region
 - the inside of the input stability circle – instability region
- $$C_S = \frac{(S_{11} - \Delta \cdot S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = -1.871 - 1.265 \cdot j$$
- $$|C_S| = 2.259$$
- $$R_S = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{11}|^2 - |\Delta|^2 \right|} = 1.325$$

ADS



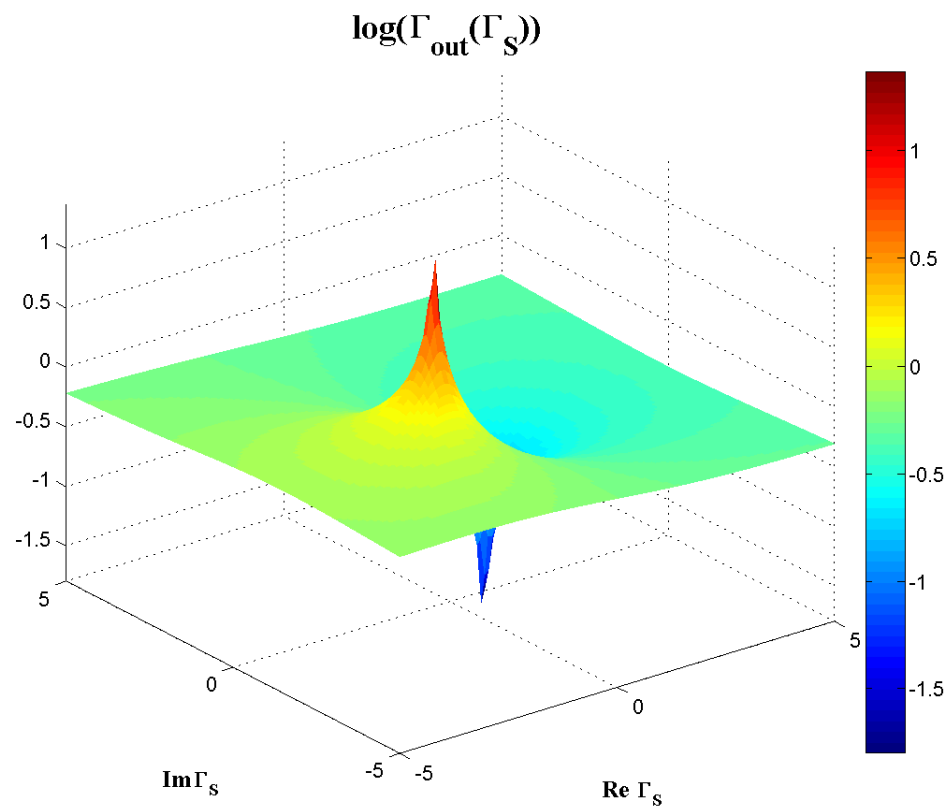
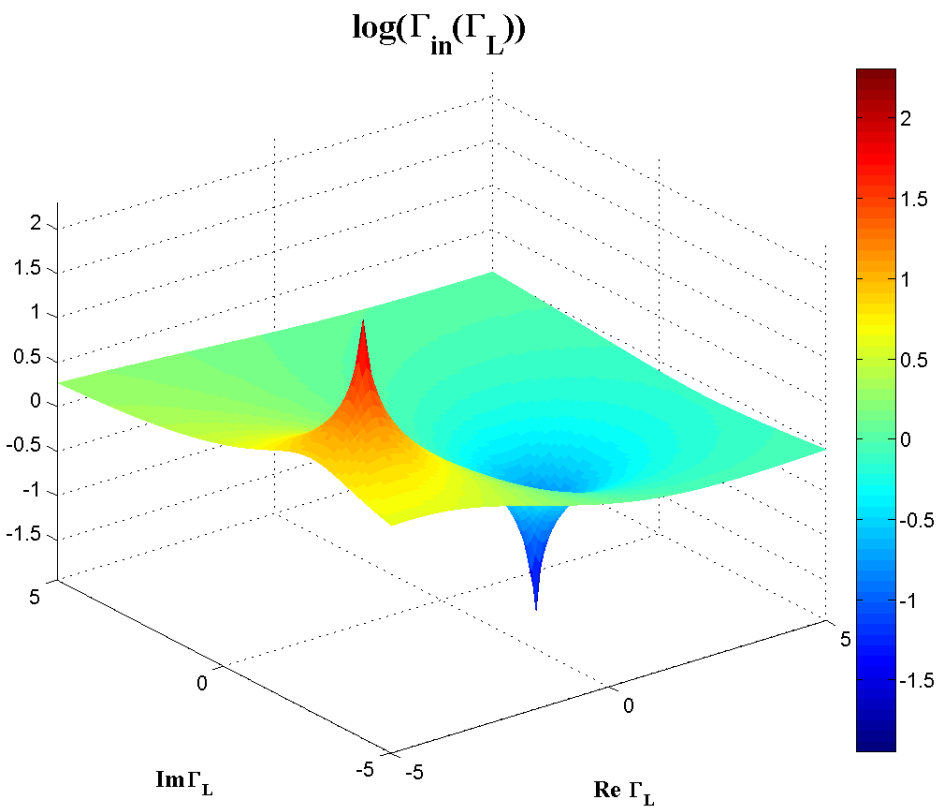
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

- High variations -> we change to z logarithmic scale



3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

- $\log_{10}|\Gamma_{in}|$, $\log_{10}|\Gamma_{out}|$

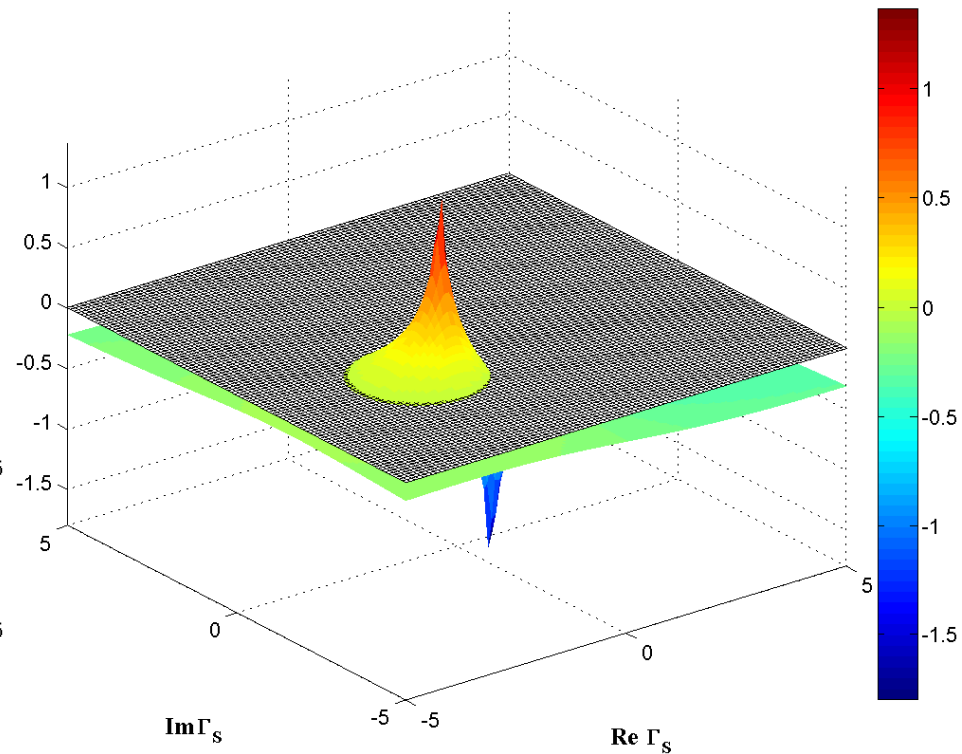
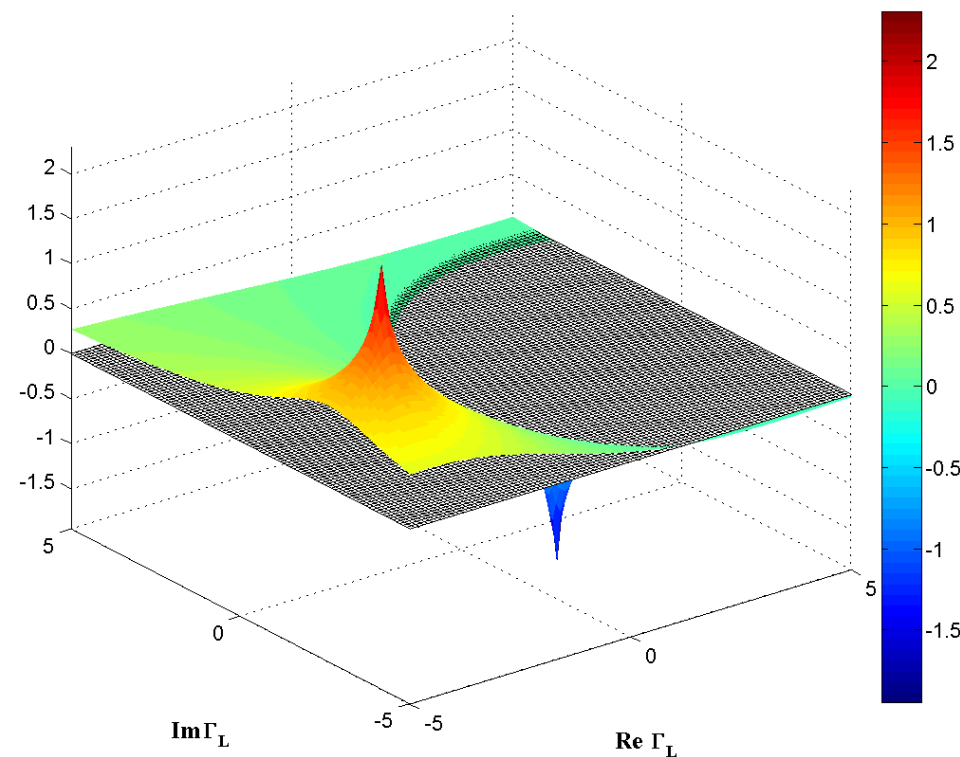


3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

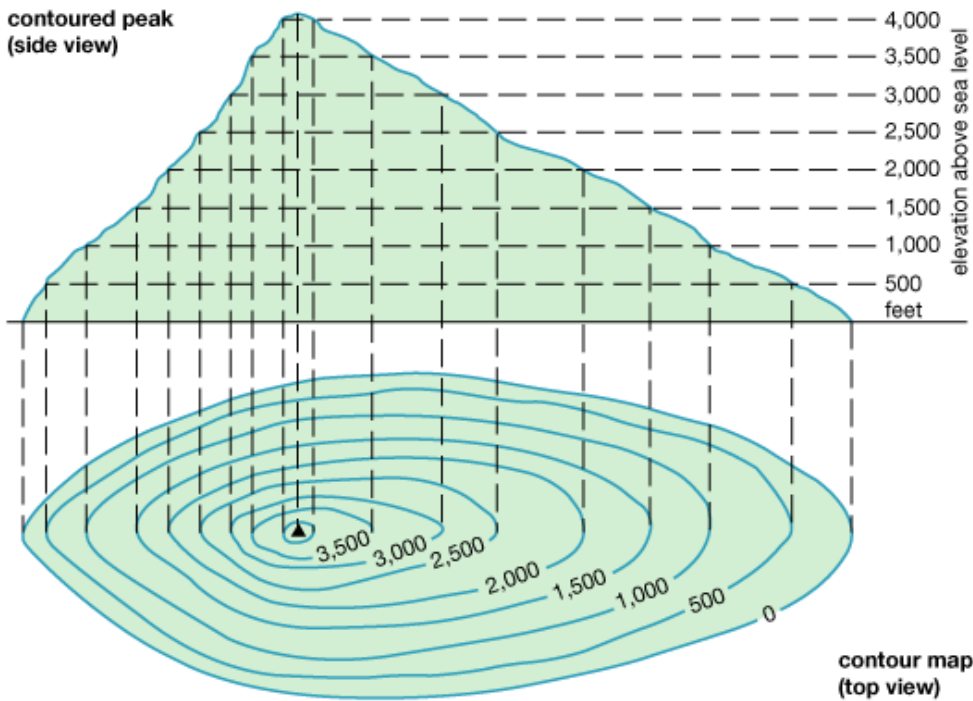
- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane $z = 0$ is a circle

$\log(\Gamma_{in}(\Gamma_L))$

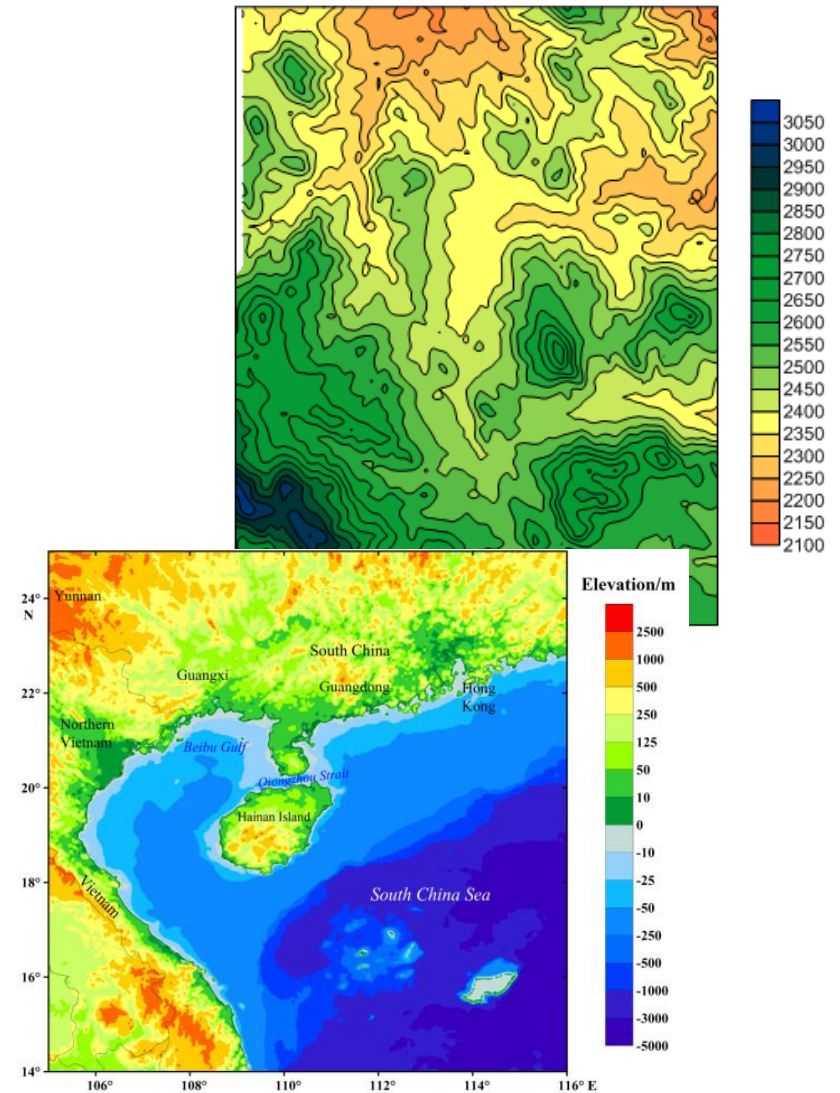
$\log(\Gamma_{out}(\Gamma_S))$



Contour map/lines



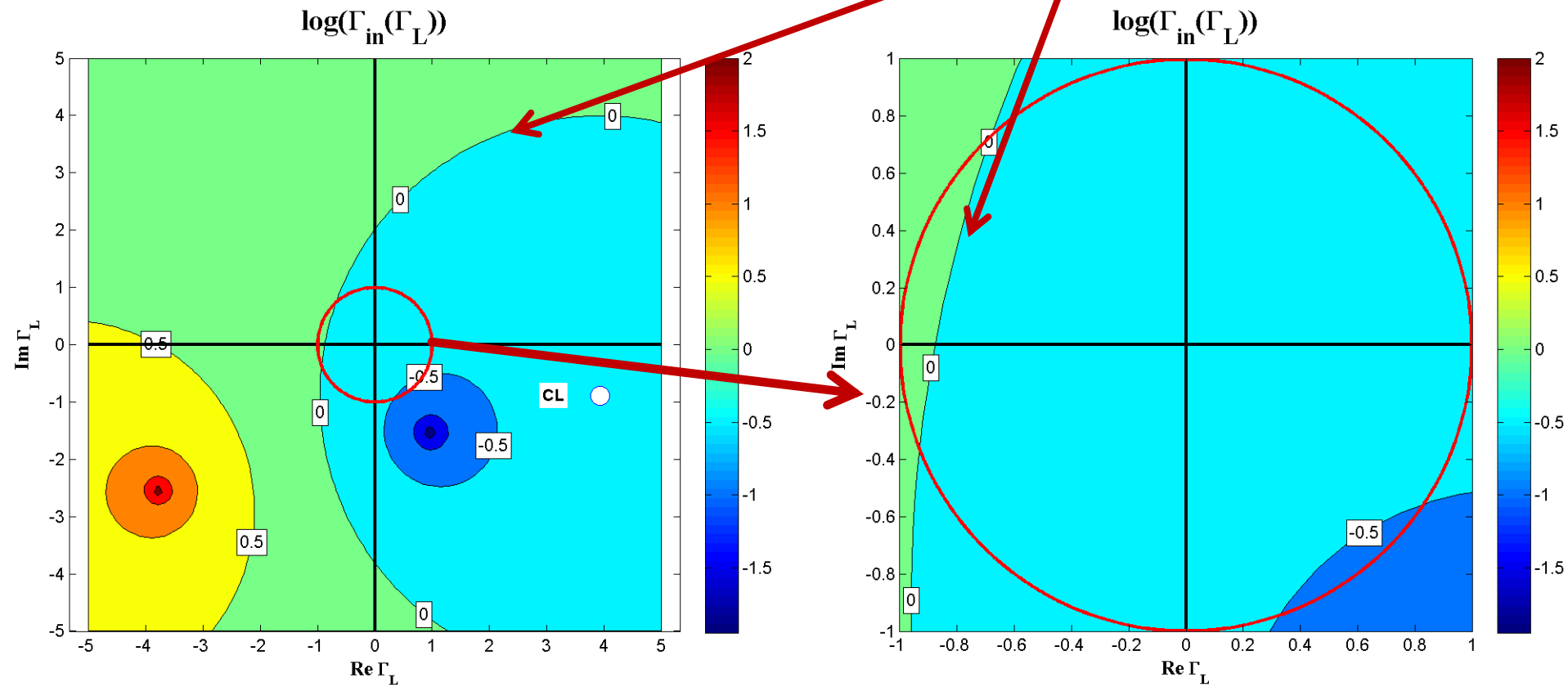
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Contour lines of $\log_{10}|\Gamma_{in}|$

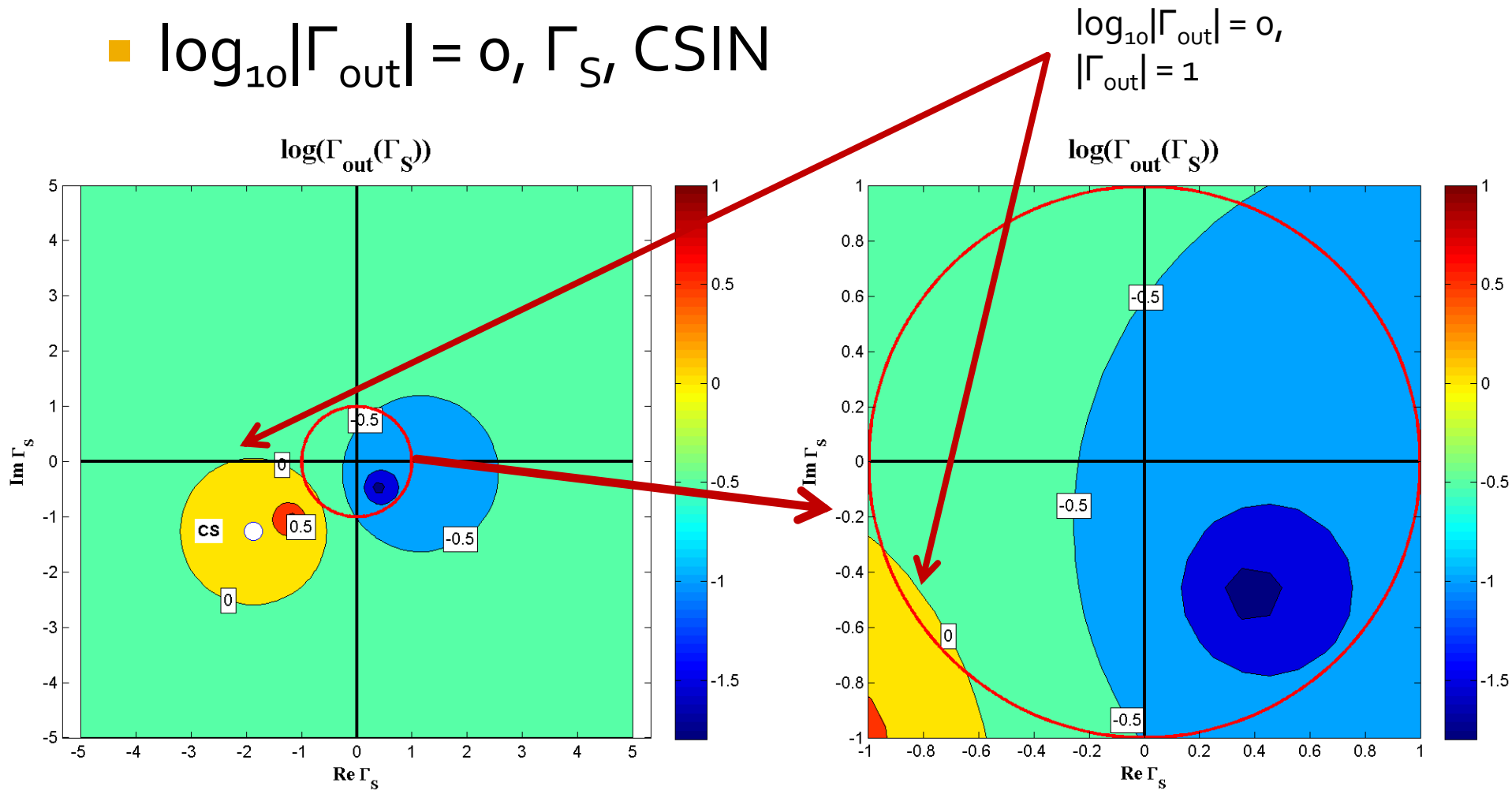
■ $\log_{10}|\Gamma_{in}| = 0, \Gamma_L, \text{CSOUT}$

$$\log_{10}|\Gamma_{in}| = 0, \\ |\Gamma_{in}| = 1$$



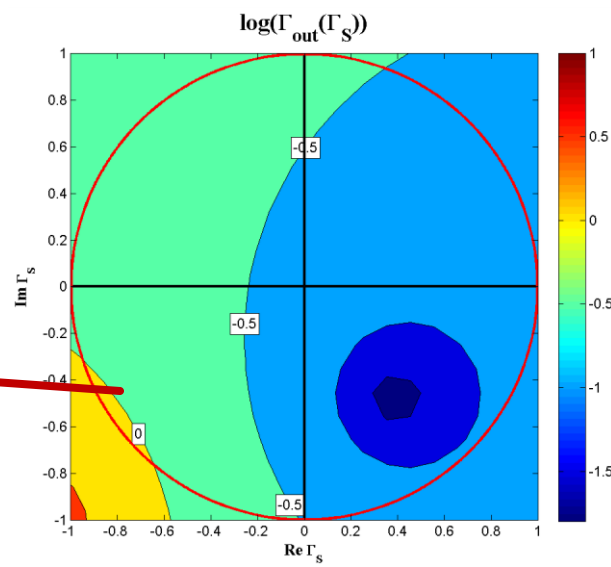
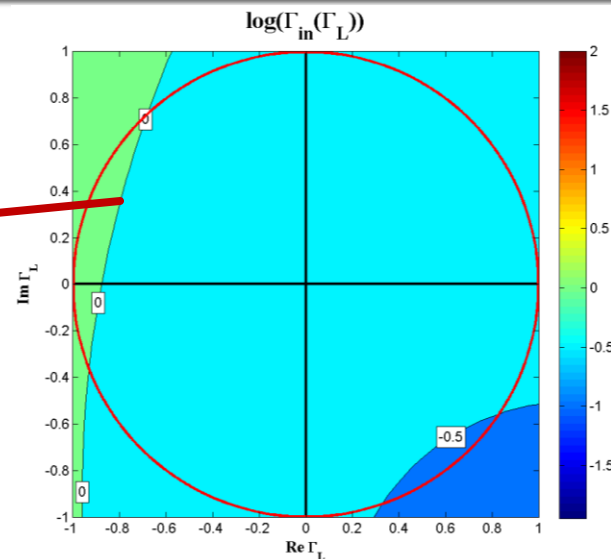
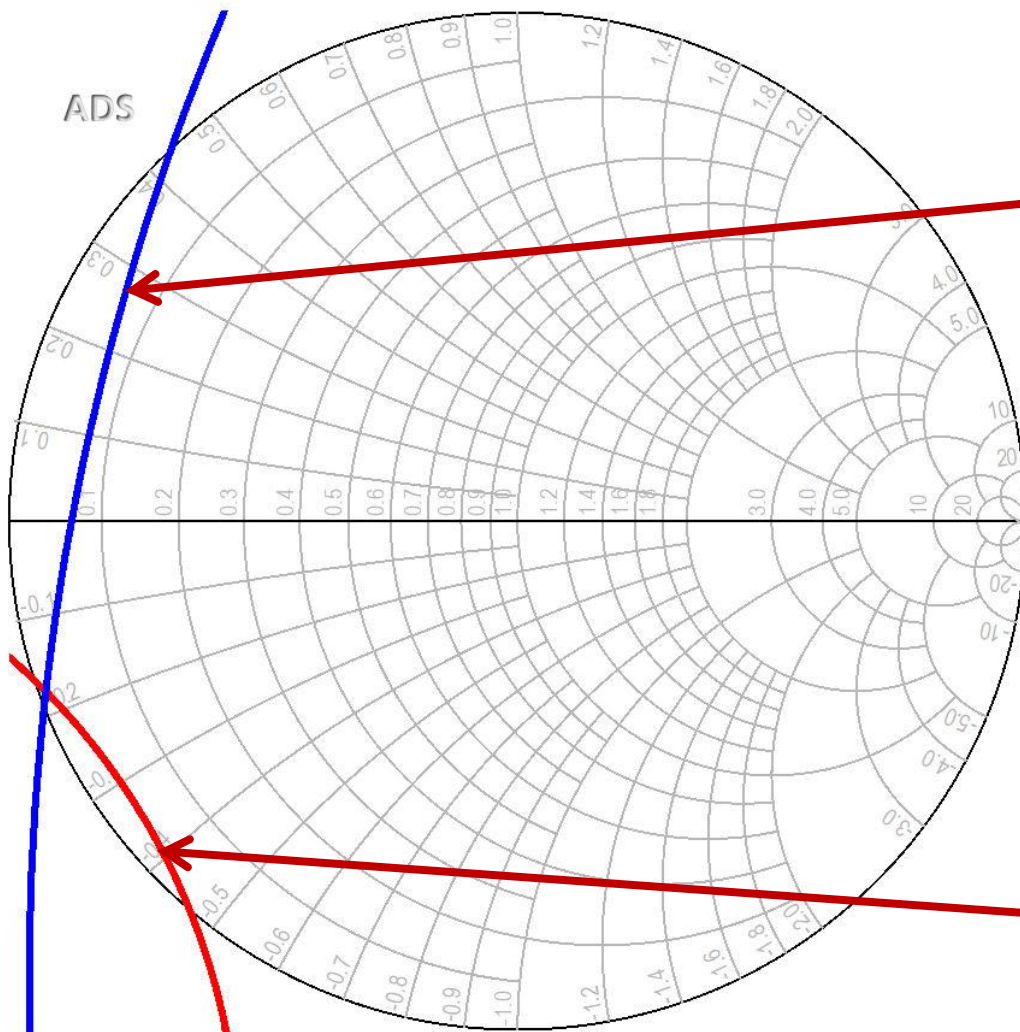
Contour lines of $\log_{10}|\Gamma_{out}|$

- $\log_{10}|\Gamma_{out}| = 0, \Gamma_S, \text{CSIN}$



CSIN, CSOUT

CSOUT
CSIN



Contact

- Microwave and Optoelectronics Laboratory
- <http://rf-opto.etti.tuiasi.ro>
- rdamian@etti.tuiasi.ro